CLASS 11 MATHS NCERT SOLUTIONS

NCERT Solutions for Class 11 Maths Chapter 12: Limits and Derivatives – Free PDF Download

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NCERT Solutions for Class 11 Maths Chapter 12, Limits and Derivatives, cover the two foundational ideas of calculus – the algebra of limits (including standard algebraic and trigonometric limits) and the derivative of a function defined using first principles, along with the algebra of derivatives and the derivatives of polynomial and trigonometric functions. This chapter has three exercises – Exercise 12.1 (limits), Exercise 12.2 (derivatives) and a Miscellaneous Exercise – and these step-by-step solutions, based on the current 2026-27 rationalised NCERT textbook, will help students build a strong base for calculus in Class 12 and competitive exams. These solutions are prepared to match the latest NCERT syllabus and exam pattern.

Exercise 12.1

Q1. Evaluate the following limit: $\lim_{x\to 3} (x + 3)$

Substituting x = 3 directly (since x + 3 is a polynomial function, continuous everywhere):
$\lim_{x\to 3} (x + 3) = 3 + 3 = \mathbf{6}$

Q2. Evaluate the following limit: $\lim_{x\to \pi} (x - 22/7)$

$\lim_{x\to \pi} (x - 22/7) = \pi - 22/7$

Q3. Evaluate the following limit: $\lim_{r\to 1} \pi r^2$

$\lim_{r\to 1} \pi r^2 = \pi(1)^2 = \boldsymbol{\pi}$

Q4. Evaluate the following limit: $\lim_{x\to 4} (4x + 3)/(x - 2)$

Since the denominator is non-zero at x = 4, we substitute directly:
$\lim_{x\to 4} (4x + 3)/(x - 2) = (16 + 3)/(4 - 2) = 19/2$

Q5. Evaluate the following limit: $\lim_{x\to -1} (x^{10} + x^5 + 1)/(x - 1)$

$\lim_{x\to -1} (x^{10} + x^5 + 1)/(x - 1) = [(-1)^{10} + (-1)^5 + 1] / (-1 - 1) = (1 - 1 + 1)/(-2) = 1/(-2) = \mathbf{-1/2}$

Q6. Evaluate the following limit: $\lim_{x\to 0} [(x + 1)^5 - 1]/x$

This is of the form 0/0. Expanding $(x + 1)^5$ by the binomial theorem:
$(x + 1)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
So $(x + 1)^5 - 1 = 5x + 10x^2 + 10x^3 + 5x^4 + x^5 = x(5 + 10x + 10x^2 + 5x^3 + x^4)$
$\lim_{x\to 0} [(x + 1)^5 - 1]/x = \lim_{x\to 0} (5 + 10x + 10x^2 + 5x^3 + x^4) = \mathbf{5}$

Q7. Evaluate the following limit: $\lim_{x\to 2} (3x^2 - x - 10)/(x^2 - 4)$

This is of the form 0/0 at x = 2, so we factorise:
$3x^2 - x - 10 = (3x + 5)(x - 2)$
$x^2 - 4 = (x - 2)(x + 2)$
$\lim_{x\to 2} (3x + 5)(x - 2) / [(x - 2)(x + 2)] = \lim_{x\to 2} (3x + 5)/(x + 2) = (6 + 5)/4 = \mathbf{11/4}$

Q8. Evaluate the following limit: $\lim_{x\to 3} (x^4 - 81)/(2x^2 - 5x - 3)$

$x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9)$
$2x^2 - 5x - 3 = (2x + 1)(x - 3)$
$\lim_{x\to3} (x + 3)(x^2 + 9)/(2x + 1) = (6)(18)/7 =$ **108/7**

Q9. Evaluate the following limit: $\lim_{x\to0} (ax + b)/(cx + 1)$

Substituting x = 0 (denominator is 1, non-zero):
$\lim_{x\to0} (ax + b)/(cx + 1) = (0 + b)/(0 + 1) =$ **b**

Q10. Evaluate the following limit: $\lim_{z\to1} (z^{1/3} - 1)/(z^{1/6} - 1)$

Using the standard result $\lim_{x\to1} (x^n - 1)/(x - 1) = n$:
Divide numerator and denominator by (z – 1):
$[(z^{1/3} - 1)/(z - 1)] \div [(z^{1/6} - 1)/(z - 1)] \to (1/3)(1)^{-2/3} \div (1/6)(1)^{-5/6} = (1/3)/(1/6) =$ **2**

Q11. Evaluate the following limit: $\lim_{x\to1} (ax^2 + bx + c)/(cx^2 + bx + a)$, $a + b + c \neq 0$

Substituting x = 1 (denominator $a + b + c \neq 0$):
$\lim_{x\to1} (ax^2 + bx + c)/(cx^2 + bx + a) = (a + b + c)/(c + b + a) =$ **1**

Q12. Evaluate the following limit: $\lim_{x\to-2} (1/x + 1/2)/(x + 2)$

$1/x + 1/2 = (2 + x)/(2x)$
So the expression $= (2 + x)/[2x(x + 2)] = 1/(2x)$, for $x \neq -2$
$\lim_{x\to-2} 1/(2x) = 1/(2 \times (-2)) =$ **–1/4**

Q13. Evaluate the following limit: $\lim_{x\to0} \sin(ax)/(bx)$

$\lim_{x\to0} \sin(ax)/(bx) = (a/b) \cdot \lim_{x\to0} \sin(ax)/(ax) = (a/b)(1) =$ **a/b**

Q14. Evaluate the following limit: $\lim_{x\to0} \sin(ax)/\sin(bx)$, $a, b \neq 0$

$\lim_{x\to0} [\sin(ax)/(ax)] \cdot [ax/bx] \cdot [bx/\sin(bx)] = 1 \cdot (a/b) \cdot 1 =$ **a/b**

Q15. Evaluate the following limit: $\lim_{x\to\pi} \sin(\pi - x)/[\pi(\pi - x)]$

Let $t = \pi - x$, so $t \to 0$ as $x \to \pi$:
$\lim_{t\to0} \sin t/(\pi t) = (1/\pi) \cdot \lim_{t\to0} (\sin t)/t =$ **1/π**

Q16. Evaluate the following limit: $\lim_{x\to0} \cos x/(\pi - x)$

$\lim_{x\to0} \cos x/(\pi - x) = \cos 0/(\pi - 0) =$ **1/π**

Q17. Evaluate the following limit: $\lim_{x\to0} (\cos 2x - 1)/(\cos x - 1)$

Using $\cos 2x - 1 = -2\sin^2 x$ and $\cos x - 1 = -2\sin^2(x/2)$:
$= \sin^2 x / \sin^2(x/2) = [2 \sin(x/2) \cos(x/2)]^2 / \sin^2(x/2) = 4\cos^2(x/2)$

$lim_{x\to0}$ $4\cos^2(x/2) = 4(1)^2 =$ **4**

Q18. Evaluate the following limit: $lim_{x\to0}$ (ax + x cos x)/(b sin x)

= $lim_{x\to0}$ x(a + cos x)/(b sin x) = $lim_{x\to0}$ [(a + cos x)/b] · [x/sin x] = [(a + 1)/b] · 1 = **(a + 1)/b**

Q19. Evaluate the following limit: $lim_{x\to0}$ x sec x

$lim_{x\to0}$ x sec x = 0 × sec 0 = 0 × 1 = **0**

Q20. Evaluate the following limit: $lim_{x\to0}$ (sin ax + bx)/(ax + sin bx), a, b, a + b ≠ 0

Dividing numerator and denominator by x:
= [a · (sin ax/ax) + b] / [a + b · (sin bx/bx)] → (a · 1 + b)/(a + b · 1) = (a + b)/(a + b) = **1**

Q21. Evaluate the following limit: $lim_{x\to0}$ (cosec x – cot x)

cosec x – cot x = (1 – cos x)/sin x = $2\sin^2(x/2)$ / [2 sin(x/2)cos(x/2)] = tan(x/2)
$lim_{x\to0}$ tan(x/2) = **0**

Q22. Evaluate the following limit: $lim_{x\to\pi/2}$ tan 2x/(x – π/2)

Let t = x – π/2, so x = t + π/2 and t → 0:
tan 2x = tan(2t + π) = tan 2t (period π)
$lim_{t\to0}$ tan 2t/t = 2 · $lim_{t\to0}$ tan 2t/(2t) = 2 × 1 = **2**

Q23. Find $lim_{x\to0}$ f(x) and $lim_{x\to1}$ f(x), where f(x) = 2x + 3, x ≤ 0 and f(x) = 3(x + 1), x > 0

At x → 0:
Left-hand limit: $lim_{x\to0^-}$ (2x + 3) = 3
Right-hand limit: $lim_{x\to0^+}$ 3(x + 1) = 3(1) = 3
Since LHL = RHL = 3, $lim_{x\to0}$ f(x) = **3**

At x → 1: (x = 1 lies in the region x > 0, and 3(x+1) is continuous)
$lim_{x\to1}$ f(x) = 3(1 + 1) = **6**

Q24. Find $lim_{x\to1}$ f(x), where f(x) = x^2 – 1, x ≤ 1 and f(x) = $-x^2$ – 1, x > 1

Left-hand limit: $lim_{x\to1^-}$ (x^2 – 1) = 1 – 1 = 0
Right-hand limit: $lim_{x\to1^+}$ ($-x^2$ – 1) = –1 – 1 = –2
Since LHL ≠ RHL, **the limit does not exist.**

Q25. Evaluate $lim_{x\to0}$ f(x), where f(x) = |x|/x, x ≠ 0 and f(0) = 0

Left-hand limit (x Right-hand limit (x > 0): |x|/x = x/x = 1
Since LHL ≠ RHL, **the limit does not exist.**

Q26. Find $lim_{x\to0}$ f(x), where f(x) = x/|x|, x ≠ 0 and f(0) = 0

Left-hand limit (x Right-hand limit ($x > 0$): $x/|x| = x/x = 1$
Since LHL ≠ RHL, **the limit does not exist.**

Q27. Find $\lim_{x\to 5} f(x)$, where $f(x) = |x| - 5$

Near $x = 5$, $|x| = x$, so f is continuous there:
$\lim_{x\to 5} (|x| - 5) = 5 - 5 = \mathbf{0}$

Q28. Suppose f(x) = a + bx for x 1. If $\lim_{x\to 1} f(x) = f(1)$, what are the possible values of a and b?

$f(1) = 4$.
Left-hand limit: $\lim_{x\to 1^-} (a + bx) = a + b$
Right-hand limit: $\lim_{x\to 1^+} (b - ax) = b - a$
For the limit to exist and equal $f(1) = 4$: $a + b = 4$ and $b - a = 4$.
Adding: $2b = 8$ ⍰ $b = 4$. Then $a = 4 - b = 0$.
So $a = 0$, $b = 4$.

Q29. Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define $f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$. What is $\lim_{x\to a_1} f(x)$? For some $a \neq a_1, a_2, \ldots, a_n$, compute $\lim_{x\to a} f(x)$.

Since f is a polynomial function, it is continuous everywhere, so the limit equals the function value.
$\lim_{x\to a_1} f(x) = f(a_1) = (a_1 - a_1)(a_1 - a_2)\ldots(a_1 - a_n) = 0$ (the first factor is zero)

For $a \neq a_1, a_2, \ldots, a_n$:
$\lim_{x\to a} f(x) = f(a) = (a - a_1)(a - a_2)\ldots(a - a_n)$

Q30. If $f(x) = 0$ for $x = 0$ and $f(x) = |x| - 1$ for $x > 0$, for what value(s) of a does $\lim_{x\to a} f(x)$ exist?

The function is defined only for $x \geq 0$.
For $a > 0$: near such a point $f(x) = |x| - 1 = x - 1$, which is continuous, so the left-hand and right-hand limits both equal $a - 1$. The limit exists and equals $a - 1$.
For $a = 0$: since f is not defined for x Conclusion: $\lim_{x\to a} f(x)$ exists for every $a > 0$, and does not exist at $a = 0$.

Q31. If the function f(x) satisfies $\lim_{x\to 1} [f(x) - 2]/(x^2 - 1) = \pi$, evaluate $\lim_{x\to 1} f(x)$.

Since the denominator $x^2 - 1 \to 0$ as $x \to 1$, and the overall limit is the finite value π, the numerator must also → 0 (otherwise the limit would be undefined or infinite).
So $\lim_{x\to 1} [f(x) - 2] = 0$, which gives $\lim_{x\to 1} f(x) = 2$

Q32. If $f(x) = mx^2 + n$ for x 1. For what integers m and n do both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$

exist?

At $x = 0$:
Left-hand limit: $\lim_{x\to 0^-} (mx^2 + n) = n$
Right-hand limit: $\lim_{x\to 0^+} (nx + m) = m$
For the limit to exist: $n = m$

At $x = 1$:
Left-hand limit: $\lim_{x\to 1^-} (nx + m) = n + m$
Right-hand limit: $\lim_{x\to 1^+} (nx^3 + m) = n + m$
These are automatically equal for any m, n.

Conclusion: both limits exist exactly when $m = n$ (m, n any integers with $m = n$).

Exercise 12.2

Q1. Find the derivative of $x^2 – 2$ at $x = 10$.

$f(x) = x^2 – 2$, so $f'(x) = 2x$ (by the power rule).
$f'(10) = 2(10) = 20$

Q2. Find the derivative of x at $x = 1$.

$f(x) = x$, so $f'(x) = 1$.
$f'(1) = 1$

Q3. Find the derivative of 99x at $x = 100$.

$f(x) = 99x$, so $f'(x) = 99$.
$f'(100) = 99$

Q4. Find the derivative of the following functions from first principle.

$x^3 – 27$
$f(x) = x^3 – 27$
$f'(x) = \lim_{h\to 0} [(x + h)^3 – 27 – (x^3 – 27)]/h = \lim_{h\to 0} [(x + h)^3 – x^3]/h$
$= \lim_{h\to 0} [3x^2h + 3xh^2 + h^3]/h = \lim_{h\to 0} (3x^2 + 3xh + h^2) = 3x^2$

$(x – 1)(x – 2)$
$f(x) = x^2 – 3x + 2$
$f(x + h) – f(x) = [(x+h)^2 – 3(x+h) + 2] – [x^2 – 3x + 2] = 2xh + h^2 – 3h = h(2x + h – 3)$
$f'(x) = \lim_{h\to 0} (2x + h – 3) = 2x – 3$

$1/x^2$
$f(x + h) – f(x) = 1/(x+h)^2 – 1/x^2 = [x^2 – (x+h)^2] / [x^2(x+h)^2] = –h(2x + h) / [x^2(x+h)^2]$
$f'(x) = \lim_{h\to 0} –(2x + h) / [x^2(x+h)^2] = –2x/x^4 = –2/x^3$

$(x + 1)/(x - 1)$
$f(x + h) - f(x) = (x+h+1)/(x+h-1) - (x+1)/(x-1)$. Combining over a common denominator, the numerator simplifies to $-2h$, so:
$f(x+h) - f(x) = -2h / [(x + h - 1)(x - 1)]$
$f'(x) = \lim_{h\to 0} -2 / [(x + h - 1)(x - 1)] = -2/(x - 1)^2$

Q5. For the function $f(x) = x^{100}/100 + x^{99}/99 + \dots + x^2/2 + x + 1$, prove that $f'(1) = 100 f'(0)$.

Differentiating term by term (using $d/dx(x^n/n) = x^{n-1}$):
$f'(x) = x^{99} + x^{98} + \dots + x + 1$ (a sum of 100 terms)
$f'(1) = 1 + 1 + \dots + 1$ (100 terms) $= 100$
$f'(0) = 0 + 0 + \dots + 0 + 1 = 1$ (only the constant term survives)
So $f'(1) = 100 = 100 \times 1 = 100 f'(0)$. Hence proved.

Q6. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n$ for a fixed real number a.

Differentiating each term using the power rule:
$f'(x) = nx^{n-1} + (n-1)ax^{n-2} + (n-2)a^2x^{n-3} + \dots + a^{n-1}$

Q7. For some constants a and b, find the derivative of:

$(x - a)(x - b)$
$= x^2 - (a+b)x + ab$ ⍰ $f'(x) = 2x - (a + b)$

$(ax^2 + b)^2$
Using the product/power pattern, $f'(x) = 2(ax^2+b) \cdot 2ax = 4ax(ax^2 + b)$

$(x - a)/(x - b)$
By the quotient rule: $f'(x) = [(1)(x-b) - (x-a)(1)]/(x-b)^2 = (a - b)/(x - b)^2$

Q8. Find the derivative of $(x^n - a^n)/(x - a)$ for some constant a.

Using $x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \dots + a^{n-1})$, the function simplifies to:
$f(x) = x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-1}$
Differentiating term by term:
$f'(x) = (n-1)x^{n-2} + (n-2)ax^{n-3} + \dots + a^{n-2}$

Q9. Find the derivative of:

$2x - 3/4$
$f'(x) = 2$

$(5x^3 + 3x - 1)(x - 1)$
Product rule with $u = 5x^3+3x-1$ ($u' = 15x^2+3$), $v = x-1$ ($v' = 1$):

$f'(x) = (15x^2+3)(x-1) + (5x^3+3x-1)(1) = 15x^3-15x^2+3x-3+5x^3+3x-1$
$= 20x^3 - 15x^2 + 6x - 4$

$x^{-3}(5 + 3x)$
$= 5x^{-3} + 3x^{-2}$ ⍰ $f'(x) = -15x^{-4} - 6x^{-3} = -15/x^4 - 6/x^3$

$x^5(3 - 6x^{-9})$
$= 3x^5 - 6x^{-4}$ ⍰ $f'(x) = 15x^4 + 24x^{-5} = 15x^4 + 24/x^5$

$x^{-4}(3 - 4x^{-5})$
$= 3x^{-4} - 4x^{-9}$ ⍰ $f'(x) = -12x^{-5} + 36x^{-10}$

$2/(x + 1) - x^2/(3x - 1)$
$d/dx[2/(x+1)] = -2/(x+1)^2$
$d/dx[x^2/(3x-1)]$ by quotient rule $= [2x(3x-1) - x^2(3)]/(3x-1)^2 = (3x^2-2x)/(3x-1)^2$
$f'(x) = -2/(x+1)^2 - (3x^2 - 2x)/(3x - 1)^2$

Q10. Find the derivative of cos x from first principle.

$f(x) = \cos x$
$f'(x) = \mathit{lim}_{h\to 0}\, [\cos(x+h) - \cos x]/h$
Using $\cos A - \cos B = -2 \sin[(A+B)/2] \sin[(A-B)/2]$:
$\cos(x+h) - \cos x = -2 \sin(x + h/2) \sin(h/2)$
$f'(x) = \mathit{lim}_{h\to 0}\, -\sin(x + h/2) \cdot [\sin(h/2)/(h/2)] = -\sin(x) \times 1 = -\sin x$

Q11. Find the derivative of the following functions:

$\sin x \cos x$
By the product rule: $(\sin x)'\cos x + \sin x(\cos x)' = \cos^2 x - \sin^2 x = \cos 2x$

$\sec x$
Using the quotient rule on $1/\cos x$: $f'(x) = [0 \cdot \cos x - 1 \cdot (-\sin x)]/\cos^2 x = \sin x/\cos^2 x = \sec x \tan x$

$5 \sec x + 4 \cos x$
$f'(x) = 5 \sec x \tan x - 4 \sin x$

$\operatorname{cosec} x$
Using the quotient rule on $1/\sin x$: $f'(x) = [0 \cdot \sin x - 1 \cdot \cos x]/\sin^2 x = -\cos x/\sin^2 x = -\operatorname{cosec} x \cot x$

$3 \cot x + 5 \operatorname{cosec} x$
$f'(x) = -3 \operatorname{cosec}^2 x - 5 \operatorname{cosec} x \cot x$

$5 \sin x - 6 \cos x + 7$
$f'(x) = 5 \cos x + 6 \sin x$

$2 \tan x - 7 \sec x$
$f'(x) = 2 \sec^2 x - 7 \sec x \tan x$

Miscellaneous Exercise

Q1. Find the derivative of the following functions from first principle (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$-x$
$f'(x) = \lim_{h\to 0} [-(x+h) - (-x)]/h = \lim_{h\to 0} (-h)/h = -1$

$(-x)^{-1} = -1/x$
$f'(x) = \lim_{h\to 0} [-1/(x+h) + 1/x]/h = \lim_{h\to 0} [h/(x(x+h))]/h = \lim_{h\to 0} 1/[x(x+h)] = 1/x^2$

$\sin(x + 1)$
$f'(x) = \lim_{h\to 0} [\sin(x+h+1) - \sin(x+1)]/h$. Using $\sin A - \sin B = 2 \cos[(A+B)/2] \sin[(A-B)/2]$:
$= \lim_{h\to 0} 2 \cos(x+1+h/2) \sin(h/2) / h = \cos(x+1) \times 1 = \cos(x + 1)$

$\cos(x - \pi/8)$
By the same method as (iii): $f'(x) = -\sin(x - \pi/8)$

Q2. Find the derivative of (x + a).

$f'(x) = 1$

Q3. Find the derivative of (px + q)(r/x + s).

Product rule with $u = px+q$ ($u'=p$), $v = rx^{-1}+s$ ($v'=-rx^{-2}$):
$f'(x) = p(r/x + s) + (px+q)(-r/x^2) = pr/x + ps - pr/x - qr/x^2$
$= ps - qr/x^2$

Q4. Find the derivative of $(ax + b)(cx + d)^2$.

Product rule with $u = ax+b$ ($u'=a$), $v = (cx+d)^2$ ($v'=2c(cx+d)$):
$f'(x) = a(cx+d)^2 + (ax+b) \cdot 2c(cx+d) = (cx+d)[a(cx+d) + 2c(ax+b)]$
$= (cx + d)(3acx + ad + 2bc)$

Q5. Find the derivative of (ax + b)/(cx + d).

Quotient rule with $u = ax+b$ ($u'=a$), $v = cx+d$ ($v'=c$):
$f'(x) = [a(cx+d) - c(ax+b)]/(cx+d)^2 = (ad - bc)/(cx + d)^2$

Q6. Find the derivative of $(1 + 1/x)/(1 - 1/x)$.

Multiplying numerator and denominator by x: $f(x) = (x + 1)/(x - 1)$
By the quotient rule: $f'(x) = [(x-1) - (x+1)]/(x-1)^2 = -2/(x - 1)^2$

Q7. Find the derivative of $1/(ax^2 + bx + c)$.

$f(x) = (ax^2+bx+c)^{-1}$ ⍰ $f'(x) = -(2ax + b)/(ax^2 + bx + c)^2$

Q8. Find the derivative of $(ax + b)/(px^2 + qx + r)$.

Quotient rule with $u = ax+b$ ($u'=a$), $v = px^2+qx+r$ ($v'=2px+q$):
$f'(x) = [a(px^2+qx+r) - (ax+b)(2px+q)]/(px^2+qx+r)^2$
Expanding the numerator: $apx^2+aqx+ar - (2apx^2+aqx+2bpx+bq) = -apx^2 - 2bpx + (ar - bq)$
$f'(x) = [-apx^2 - 2bpx + (ar - bq)] / (px^2 + qx + r)^2$

Q9. Find the derivative of $(px^2 + qx + r)/(ax + b)$.

Quotient rule with $u = px^2+qx+r$ ($u'=2px+q$), $v = ax+b$ ($v'=a$):
$f'(x) = [(2px+q)(ax+b) - a(px^2+qx+r)]/(ax+b)^2$
Expanding the numerator: $2apx^2+2bpx+aqx+bq - apx^2-aqx-ar = apx^2+2bpx+(bq-ar)$
$f'(x) = [apx^2 + 2bpx + (bq - ar)] / (ax + b)^2$

Q10. Find the derivative of $a/x^4 - b/x^2 + \cos x$.

$f(x) = ax^{-4} - bx^{-2} + \cos x$
$f'(x) = -4a/x^5 + 2b/x^3 - \sin x$

Q11. Find the derivative of $4\sqrt{x} - 2$.

$f(x) = 4x^{1/2} - 2$ ⍰ $f'(x) = 4 \times (1/2)x^{-1/2} = 2/\sqrt{x}$

Q12. Find the derivative of $(ax + b)^n$.

Writing $t = ax + b$ and following the same first-principle pattern used for x^n (Theorem 6), with the increment in t equal to ah when x increases by h:
$f'(x) = na(ax + b)^{n-1}$

Q13. Find the derivative of $(ax + b)^n(cx + d)^m$.

Product rule with $u = (ax+b)^n$ ($u' = na(ax+b)^{n-1}$), $v = (cx+d)^m$ ($v' = mc(cx+d)^{m-1}$):
$f'(x) = na(ax+b)^{n-1}(cx+d)^m + mc(ax+b)^n(cx+d)^{m-1}$
$= (ax+b)^{n-1}(cx+d)^{m-1} [na(cx+d) + mc(ax+b)]$

Q14. Find the derivative of sin(x + a).

By the same method as Q1(iii) above: $f'(x) = \cos(x + a)$

Q15. Find the derivative of cosec x cot x.

Product rule with $u = \text{cosec } x$ ($u' = -\text{cosec } x \cot x$), $v = \cot x$ ($v' = -\text{cosec}^2 x$):
$f'(x) = (-\text{cosec } x \cot x)(\cot x) + (\text{cosec } x)(-\text{cosec}^2 x) = -\text{cosec } x \cot^2 x - \text{cosec}^3 x$
$= -\text{cosec } x (\cot^2 x + \text{cosec}^2 x)$

Q16. Find the derivative of cos x/(1 + sin x).

Quotient rule with $u = \cos x$ ($u' = -\sin x$), $v = 1+\sin x$ ($v' = \cos x$):
$f'(x) = [-\sin x(1+\sin x) - \cos^2 x]/(1+\sin x)^2 = [-\sin x - \sin^2 x - \cos^2 x]/(1+\sin x)^2 = [-\sin x - 1]/(1+\sin x)^2$
$= -1/(1 + \sin x)$

Q17. Find the derivative of (sin x + cos x)/(sin x − cos x).

Quotient rule with $u = \sin x+\cos x$ ($u' = \cos x-\sin x$), $v = \sin x-\cos x$ ($v' = \cos x+\sin x$):
Numerator $= (\cos x-\sin x)(\sin x-\cos x) - (\sin x+\cos x)(\cos x+\sin x) = -(1 - \sin 2x) - (1 + \sin 2x) = -2$
$f'(x) = -2/(\sin x - \cos x)^2$

Q18. Find the derivative of (sec x − 1)/(sec x + 1).

Quotient rule with $u = \sec x-1$ ($u' = \sec x \tan x$), $v = \sec x+1$ ($v' = \sec x \tan x$):
Numerator $= \sec x \tan x(\sec x+1) - (\sec x-1)\sec x \tan x = \sec x \tan x[(\sec x+1)-(\sec x-1)] = 2 \sec x \tan x$
$f'(x) = 2 \sec x \tan x / (\sec x + 1)^2$

Q19. Find the derivative of $\sin^n x$.

Writing $\sin^n x = \sin x \cdot \sin^{n-1} x$ and applying the product rule repeatedly (i.e. by induction on n), the power of a function differentiates the same way as the power rule:
$f'(x) = n \sin^{n-1} x \cos x$

Q20. Find the derivative of (a + b sin x)/(c + d cos x).

Quotient rule with $u = a+b \sin x$ ($u' = b \cos x$), $v = c+d \cos x$ ($v' = -d \sin x$):
$f'(x) = [b \cos x(c+d \cos x) + d \sin x(a+b \sin x)]/(c+d \cos x)^2$
$= [bc \cos x + bd \cos^2 x + ad \sin x + bd \sin^2 x]/(c+d \cos x)^2$
$= (bd + bc \cos x + ad \sin x) / (c + d \cos x)^2$

Q21. Find the derivative of $\sin(x + a)/\cos x$.

Quotient rule with $u = \sin(x+a)$ ($u' = \cos(x+a)$), $v = \cos x$ ($v' = -\sin x$):
$f'(x) = [\cos(x+a)\cos x + \sin(x+a)\sin x]/\cos^2 x = \cos(x+a-x)/\cos^2 x$
$= \cos a \cdot \sec^2 x$

Q22. Find the derivative of $x^4(5 \sin x - 3 \cos x)$.

Product rule with $u = x^4$ ($u' = 4x^3$), $v = 5 \sin x-3 \cos x$ ($v' = 5 \cos x+3 \sin x$):
$f'(x) = 4x^3(5 \sin x-3 \cos x) + x^4(5 \cos x+3 \sin x)$
$= 20x^3 \sin x - 12x^3 \cos x + 5x^4 \cos x + 3x^4 \sin x$

Q23. Find the derivative of $(x^2 + 1) \cos x$.

Product rule with $u = x^2+1$ ($u' = 2x$), $v = \cos x$ ($v' = -\sin x$):
$f'(x) = 2x \cos x - (x^2 + 1) \sin x$

Q24. Find the derivative of $(ax^2 + \sin x)(p + q \cos x)$.

Product rule with $u = ax^2+\sin x$ ($u' = 2ax+\cos x$), $v = p+q \cos x$ ($v' = -q \sin x$):
$f'(x) = (2ax + \cos x)(p + q \cos x) - q \sin x(ax^2 + \sin x)$

Q25. Find the derivative of $(x + \cos x)(x - \tan x)$.

Product rule with $u = x+\cos x$ ($u' = 1-\sin x$), $v = x-\tan x$ ($v' = 1-\sec^2 x = -\tan^2 x$):
$f'(x) = (1 - \sin x)(x - \tan x) - \tan^2 x\ (x + \cos x)$

Q26. Find the derivative of $(4x + 5 \sin x)/(3x + 7 \cos x)$.

Quotient rule with $u = 4x+5 \sin x$ ($u' = 4+5 \cos x$), $v = 3x+7 \cos x$ ($v' = 3-7 \sin x$):
Numerator $= (4+5\cos x)(3x+7\cos x) - (4x+5\sin x)(3-7\sin x)$
Expanding and simplifying using $\sin^2 x + \cos^2 x = 1$:
$= 35 + 28 \cos x - 15 \sin x + x(15 \cos x + 28 \sin x)$
$f'(x) = [35 + 28 \cos x - 15 \sin x + x(15 \cos x + 28 \sin x)] / (3x + 7 \cos x)^2$

Q27. Find the derivative of $x^2 \cos(\pi/4)/\sin x$.

Here $\cos(\pi/4) = 1/\sqrt{2}$ is simply a constant multiplier. Using the quotient rule on $x^2/\sin x$ with $u = x^2$ ($u'=2x$), $v = \sin x$ ($v'=\cos x$):
$d/dx[x^2/\sin x] = [2x \sin x - x^2 \cos x]/\sin^2 x$
$f'(x) = (1/\sqrt{2}) \cdot [2x \sin x - x^2 \cos x] / \sin^2 x$

Q28. Find the derivative of $x/(1 + \tan x)$.

Quotient rule with u = x (u′=1), v = 1+tan x (v′=$\sec^2$x):
f′(x) = [(1 + tan x) − x $\sec^2$x] / $(1 + \tan x)^2$

Q29. Find the derivative of (x + sec x)(x − tan x).

Product rule with u = x+sec x (u′ = 1+sec x tan x), v = x−tan x (v′ = −$\tan^2$x):
f′(x) = (1 + sec x tan x)(x − tan x) − $\tan^2$x (x + sec x)

Q30. Find the derivative of x/$\sin^n$ x.

Product rule with u = x (u′=1), v = $\sin^{-n}$x (v′ = −n $\sin^{-n-1}$x cos x, using Q19's result):
f′(x) = $\sin^{-n}$x − nx cos x $\sin^{-n-1}$x = (1/$\sin^n$x) − (nx cos x)/$\sin^{n+1}$x
= (sin x − nx cos x) / $\sin^{n+1}$x

Class 11 Maths Chapter 12 – Notes and Extra Questions

- The left-hand limit is the value approached by f(x) as x approaches a point a from values less than a; the right-hand limit is the value approached from values greater than a. A limit exists at a point only when the left-hand and right-hand limits are equal.

- The algebra of limits allows the limit of a sum, difference, product, constant multiple, or quotient (with non-zero denominator limit) of two functions to be computed from the limits of the individual functions.

- For polynomial and rational functions, the limit at a point in the domain is simply the value of the function at that point; indeterminate 0/0 forms are resolved by factorising and cancelling common factors.

- Two standard trigonometric limits used throughout the chapter: $\lim_{x\to 0}$ (sin x)/x = 1 and $\lim_{x\to 0}$ (1 − cos x)/x = 0.

- The derivative of a function f at a point a is defined by first principles as f′(a) = $\lim_{h\to 0}$ [f(a+h) − f(a)]/h, provided this limit exists.

- The algebra of derivatives includes the sum/difference rule, the constant multiple rule, the Leibniz product rule (uv)′ = u′v + uv′, and the quotient rule (u/v)′ = (u′v − uv′)/v^2.

- The power rule d/dx(x^n) = nx^{n-1} holds for any positive integer n (and, more generally, for any real number n).

- Standard derivatives to remember: d/dx(sin x) = cos x, d/dx(cos x) = −sin x, d/dx(tan x) = $\sec^2$x, d/dx(cot x) = −$\csc^2$x, d/dx(sec x) = sec x tan x, d/dx(cosec x) = −cosec x cot x.

Extra practice question 1: Evaluate $\lim_{x \to 0}$ (1 – cos 4x)/x^2.

Extra practice question 2: Find the derivative of f(x) = (x^2 + 1)/(x – 3) using the quotient rule.

Extra practice question 3: Using first principles, find the derivative of f(x) = √x.

Q1. How many exercises are there in Class 11 Maths Chapter 12, Limits and Derivatives?

The current NCERT textbook (2026-27 rationalised edition) has two exercises – Exercise 12.1 (32 questions on limits) and Exercise 12.2 (11 questions on derivatives) – plus a Miscellaneous Exercise with 30 questions that mixes both topics.

Q2. What is the difference between the left-hand limit and the right-hand limit?

The left-hand limit is the value a function approaches as the input approaches a given point from values smaller than that point, while the right-hand limit is the value approached from values larger than that point. The two-sided limit at a point exists only if these two one-sided limits are equal.

Q3. What is the first-principles definition of a derivative, and is it used in the Class 11 exercises?

The derivative of f at a point a is defined as f′(a) = $\lim_{h \to 0}$ [f(a+h) – f(a)]/h. Several questions in Exercise 12.2 and the Miscellaneous Exercise explicitly ask students to differentiate functions "from first principle" using this limit definition, in addition to using the standard algebra-of-derivatives rules.

Q4. Does Chapter 12 cover the product rule and quotient rule for derivatives?

Yes. Along with the sum, difference, and constant-multiple rules, the chapter's algebra of derivatives (Theorem 5) includes the Leibnitz product rule and the quotient rule, both of which are used repeatedly in Exercise 12.2 and the Miscellaneous Exercise.