

CLASS 11

MATHS

NCERT SOLUTIONS

## NCERT Solutions for Class 11 Maths Chapter 2: Relations and Functions – Free PDF Download

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Class 11 Maths Chapter 2, Relations and Functions, builds the language that the rest of the Maths syllabus – and much of Class 12 Calculus – is written in: ordered pairs, the Cartesian product of sets, relations between two sets, and functions as a special type of relation with a unique image for every input. Be...

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Class 11 Maths Chapter 2, Relations and Functions, builds the language that the rest of the Maths syllabus – and much of Class 12 Calculus – is written in: ordered pairs, the Cartesian product of sets, relations between two sets, and functions as a special type of relation with a unique image for every input. Below you will find complete, step-by-step NCERT solutions for every exercise of this chapter – Exercise 2.1, Exercise 2.2, Exercise 2.3, and the Miscellaneous Exercise – exactly as they appear in the current rationalised (2023) NCERT textbook used for the 2026-27 session. These Class 11 Maths Chapter 2 solutions are also useful as quick revision notes before exams.

## Exercise 2.1

**Q1. If  $(x/3 + 1, y - 2/3) = (5/3, 1/3)$ , find the values of  $x$  and  $y$ .**

Two ordered pairs are equal only when their corresponding first elements are equal and their corresponding second elements are equal. So we equate the first components and the second components separately.

First components:  $x/3 + 1 = 5/3$ . Subtracting 1 from both sides,  $x/3 = 5/3 - 1 = 5/3 - 3/3 = 2/3$ . Multiplying both sides by 3,  $x = 2$ .

Second components:  $y - 2/3 = 1/3$ . Adding  $2/3$  to both sides,  $y = 1/3 + 2/3 = 3/3 = 1$ .

**Answer:**  $x = 2$  and  $y = 1$ .

**Q2. If set  $A$  has 3 elements and set  $B = \{3, 4, 5\}$ , then find the number of elements in  $(A \times B)$ .**

For any two finite sets  $P$  and  $Q$ , the number of elements in their Cartesian product is  $n(P \times Q) = n(P) \times n(Q)$ , because every element of  $P$  can be paired with every element of  $Q$  exactly once.

Here  $n(A) = 3$  and  $n(B) = 3$  (since  $B = \{3, 4, 5\}$  has 3 elements). So  $n(A \times B) = 3 \times 3 = 9$ .

**Answer:**  $A \times B$  has 9 elements.

**Q3. If  $G = \{7, 8\}$  and  $H = \{5, 4, 2\}$ , find  $G \times H$  and  $H \times G$ .**

By definition,  $P \times Q = \{(p, q) : p \in P, q \in Q\}$ . To form  $G \times H$ , pair every element of  $G$  with every element of  $H$ , keeping the element of  $G$  first.

$$G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$$

To form  $H \times G$ , pair every element of  $H$  with every element of  $G$ , keeping the element of  $H$  first.

$$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$$

Note that  $G \times H \neq H \times G$ , which shows the Cartesian product is not commutative.

**Q4. State whether each of the following statements is true or false. If the statement is false, rewrite the given statement correctly.**

**If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then  $P \times Q = \{(m, n), (n, m)\}$ .**

This is **False**. As sets, order does not matter, so  $P = \{m, n\}$  and  $Q = \{n, m\}$  are actually the same set,  $\{m, n\}$ . Therefore  $P \times Q$  must contain every ordered pair formed by taking the first element from  $P$  and the second from  $Q$ , which includes pairing  $m$  with  $m$  and  $n$  with  $n$  as well.

Correct statement: If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then  $P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$ .

**If  $A$  and  $B$  are non-empty sets, then  $A \times B$  is a non-empty set of ordered pairs  $(x, y)$  such that  $x \in A$  and  $y \in B$ .**

This is **True** – it is exactly the definition of the Cartesian product of two non-empty sets.

**If  $A = \{1, 2\}$ ,  $B = \{3, 4\}$ , then  $A \times (B \cap \emptyset) = \emptyset$ .**

This is **True**. The intersection of any set with the empty set is the empty set, so  $B \cap \emptyset = \emptyset$ . Then  $A \times \emptyset$  is also empty, because there is no element of  $\emptyset$  to pair with elements of  $A$ .

**Q5. If  $A = \{-1, 1\}$ , find  $A \times A \times A$ .**

For a set  $A$ ,  $A \times A \times A$  is the set of all ordered triples  $(a, b, c)$  with  $a, b, c \in A$ . Since  $A$  has 2 elements,  $A \times A \times A$  has  $2^3 = 8$  elements. Listing every possible combination of  $-1$  and  $1$  in three positions:

$$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

**Q6. If  $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$ . Find  $A$  and  $B$ .**

In a Cartesian product  $A \times B$ , the set  $A$  is precisely the set of all distinct first elements appearing in the ordered pairs, and  $B$  is the set of all distinct second elements. Scanning the given set, the first elements that occur are  $a$  and  $b$ , and the second elements that occur are  $x$  and  $y$ .

**Answer:**  $A = \{a, b\}$  and  $B = \{x, y\}$ .

**Q7. Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$ . Verify that**

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

First find  $B \cap C$ :  $B = \{1, 2, 3, 4\}$  and  $C = \{5, 6\}$  have no elements in common, so  $B \cap C = \emptyset$ . Hence the left side,  $A \times (B \cap C) = A \times \emptyset = \emptyset$ .

Now compute the right side.  $A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$  and  $A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$ . Every pair in  $A \times B$  has second element in  $\{1, 2, 3, 4\}$ , while every pair in  $A \times C$  has second element in  $\{5, 6\}$  – these sets of pairs share nothing in common, so  $(A \times B) \cap (A \times C) = \emptyset$ .

Since both sides equal  $\emptyset$ ,  $LHS = RHS$ . Hence verified.

**$A \times C$  is a subset of  $B \times D$**

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$ . Since  $A = \{1, 2\} \subseteq \{1, 2, 3, 4\} = B$  and  $C = \{5, 6\} \subseteq \{5, 6, 7, 8\} = D$ , every pair  $(a, c)$  with  $a \in A$  and  $c \in C$  automatically satisfies  $a \in B$  and  $c \in D$ , so it is also a member of  $B \times D$ .

Checking directly, all four pairs  $(1, 5), (1, 6), (2, 5), (2, 6)$  do appear among the elements of  $B \times D$ .

Therefore  $A \times C \subseteq B \times D$ . Hence verified.

**Q8. Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$ . Write  $A \times B$ . How many subsets will  $A \times B$  have? List them.**

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$ , so  $n(A \times B) = 4$ .

A set with  $n$  elements has  $2^n$  subsets (this follows because each element can independently be included or excluded from a subset). With  $n = 4$ , the number of subsets of  $A \times B$  is  $2^4 = 16$ .

The 16 subsets are:  $\emptyset$ ,  $\{(1, 3)\}$ ,  $\{(1, 4)\}$ ,  $\{(2, 3)\}$ ,  $\{(2, 4)\}$ ,  $\{(1, 3), (1, 4)\}$ ,  $\{(1, 3), (2, 3)\}$ ,  $\{(1, 3), (2, 4)\}$ ,  $\{(1, 4), (2, 3)\}$ ,  $\{(1, 4), (2, 4)\}$ ,  $\{(2, 3), (2, 4)\}$ ,  $\{(1, 3), (1, 4), (2, 3)\}$ ,  $\{(1, 3), (1, 4), (2, 4)\}$ ,  $\{(1, 3), (2, 3), (2, 4)\}$ ,  $\{(1, 4), (2, 3), (2, 4)\}$ , and  $\{(1, 3), (1, 4), (2, 3), (2, 4)\}$ .

**Q9. Let  $A$  and  $B$  be two sets such that  $n(A) = 3$  and  $n(B) = 2$ . If  $(x, 1), (y, 2), (z, 1)$  are in  $A \times B$ , find  $A$  and  $B$ , where  $x, y$  and  $z$  are distinct elements.**

Since  $(x, 1), (y, 2), (z, 1) \in A \times B$ , the first elements  $x, y, z$  must all belong to  $A$ , and the second elements 1 and 2 must belong to  $B$ . As  $x, y$  and  $z$  are distinct and  $n(A) = 3$ , these three elements are precisely all of  $A$ . Since only the values 1 and 2 appear as second components and  $n(B) = 2$ , these are precisely all of  $B$ .

**Answer:**  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ .

**Q10. The Cartesian product  $A \times A$  has 9 elements, among which are found  $(-1, 0)$  and  $(0, 1)$ . Find the set  $A$  and the remaining elements of  $A \times A$ .**

For any finite set  $A$ ,  $n(A \times A) = [n(A)]^2$ . We are told  $n(A \times A) = 9$ , so  $[n(A)]^2 = 9$ , giving  $n(A) = 3$ .

Every element of  $A \times A$  is of the form  $(a, a')$  where  $a$  and  $a'$  both come from  $A$ . Since  $(-1, 0) \in A \times A$ , both  $-1$  and  $0$  must belong to  $A$ ; since  $(0, 1) \in A \times A$ , both  $0$  and  $1$  must belong to  $A$ . So  $A$  contains at least  $\{-1, 0, 1\}$ , which already has 3 elements – and since  $n(A) = 3$ , this must be exactly  $A$ .

**$A = \{-1, 0, 1\}$ .**

All 9 elements of  $A \times A$  are:  $(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)$ . Removing the two given pairs,  $(-1, 0)$  and  $(0, 1)$ , the remaining 7 elements are:

$(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

## Exercise 2.2

**Q1. Let  $A = \{1, 2, 3, \dots, 14\}$ . Define a relation  $R$  from  $A$  to  $A$  by  $R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$ . Write down its domain, codomain and range.**

The condition  $3x - y = 0$  means  $y = 3x$ . Substitute each value of  $x$  from  $A$  and keep only those pairs where  $y$  also lies in  $A$  (that is,  $y \leq 14$ ):

$x = 1 \rightarrow y = 3$ ;  $x = 2 \rightarrow y = 6$ ;  $x = 3 \rightarrow y = 9$ ;  $x = 4 \rightarrow y = 12$ ;  $x = 5 \rightarrow y = 15$ , which is not in  $A$ , so we stop here.

$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Domain (set of all first elements) = {1, 2, 3, 4}. Codomain is the whole set  $A = \{1, 2, 3, \dots, 14\}$ . Range (set of all second elements actually appearing) = {3, 6, 9, 12}.

**Q2. Define a relation R on the set N of natural numbers by  $R = \{(x, y): y = x + 5, x \text{ is a natural number less than 4}; x, y \in N\}$ . Depict this relationship using roster form. Write down the domain and the range.**

The natural numbers less than 4 are 1, 2, and 3. Substituting each into  $y = x + 5$ :

$$x = 1 \rightarrow y = 6; x = 2 \rightarrow y = 7; x = 3 \rightarrow y = 8.$$

$$R = \{(1, 6), (2, 7), (3, 8)\}. \text{ Domain} = \{1, 2, 3\}. \text{ Range} = \{6, 7, 8\}.$$

**Q3.  $A = \{1, 2, 3, 5\}$  and  $B = \{4, 6, 9\}$ . Define a relation R from A to B by  $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$ . Write R in roster form.**

Check every pair  $(x, y)$  with  $x \in A, y \in B$  and test whether  $|x - y|$  is odd:

$$x = 1: |1-4| = 3 \text{ (odd, keep)}; |1-6| = 5 \text{ (odd, keep)}; |1-9| = 8 \text{ (even, reject)}.$$

$$x = 2: |2-4| = 2 \text{ (even, reject)}; |2-6| = 4 \text{ (even, reject)}; |2-9| = 7 \text{ (odd, keep)}.$$

$$x = 3: |3-4| = 1 \text{ (odd, keep)}; |3-6| = 3 \text{ (odd, keep)}; |3-9| = 6 \text{ (even, reject)}.$$

$$x = 5: |5-4| = 1 \text{ (odd, keep)}; |5-6| = 1 \text{ (odd, keep)}; |5-9| = 4 \text{ (even, reject)}.$$

$$R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

**Q4. The figure shows a relationship between the sets P and Q. Write this relation (i) in set-builder form (ii) in roster form. What are their domain and range?**

The arrow diagram in the textbook shows  $P = \{5, 6, 7\}$  and  $Q = \{3, 4, 5\}$ , with each element of P linked to the element of Q that is exactly 2 less than it:  $5 \rightarrow 3, 6 \rightarrow 4$ , and  $7 \rightarrow 5$ .

**Set-builder form:**  $R = \{(x, y): y = x - 2, x \in P\}$ , i.e.  $R = \{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$ .

**Roster form:**  $R = \{(5, 3), (6, 4), (7, 5)\}$ .

Domain of  $R = \{5, 6, 7\}$ . Range of  $R = \{3, 4, 5\}$ .

**Q5. Let  $A = \{1, 2, 3, 4, 6\}$ . Let R be the relation on A defined by  $\{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$ .**

**Write R in roster form.**

For each  $a$  in A, find every  $b$  in A that  $a$  divides exactly:

$a = 1$  divides 1, 2, 3, 4, 6;  $a = 2$  divides 2, 4, 6;  $a = 3$  divides 3, 6;  $a = 4$  divides 4;  $a = 6$  divides 6.

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$$

**Find the domain of R.**

Every element of A appears as a first element at least once, so Domain of  $R = \{1, 2, 3, 4, 6\}$ .

**Find the range of R.**

Every element of A also appears as a second element at least once, so Range of  $R = \{1, 2, 3, 4, 6\}$ .

**Q6. Determine the domain and range of the relation R defined by  $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$ .**

Substitute each x into  $x + 5$ :  $0 \rightarrow 5, 1 \rightarrow 6, 2 \rightarrow 7, 3 \rightarrow 8, 4 \rightarrow 9, 5 \rightarrow 10$ .

$R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$ . Domain of  $R = \{0, 1, 2, 3, 4, 5\}$ . Range of  $R = \{5, 6, 7, 8, 9, 10\}$ .

**Q7. Write the relation  $R = \{(x, x^3): x \text{ is a prime number less than } 10\}$  in roster form.**

The prime numbers less than 10 are 2, 3, 5, and 7. Cubing each:  $2^3 = 8, 3^3 = 27, 5^3 = 125, 7^3 = 343$ .

$R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

**Q8. Let  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ . Find the number of relations from A to B.**

A relation from A to B is any subset of  $A \times B$ . First,  $n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6$ . A set with 6 elements has  $2^6$  possible subsets, so the number of possible relations from A to B is  $2^6 = 64$ .

**Q9. Let R be the relation on Z defined by  $R = \{(a, b): a, b \in \mathbb{Z}, a - b \text{ is an integer}\}$ . Find the domain and range of R.**

Since a and b are both already integers, their difference  $a - b$  is automatically always an integer, no matter what values a and b take. So the condition is satisfied for every pair of integers, and R is simply all of  $\mathbb{Z} \times \mathbb{Z}$ .

Domain of  $R = \mathbb{Z}$ . Range of  $R = \mathbb{Z}$ .

## Exercise 2.3

**Q1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.**

$\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

Every first element (2, 5, 8, 11, 14, 17) is distinct and has exactly one image, so this relation **is a function** (having the same image, 1, for different inputs is allowed – a function only forbids one input having two different outputs).

Domain =  $\{2, 5, 8, 11, 14, 17\}$ , Range =  $\{1\}$ .

$\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

Every first element is distinct, so this relation **is a function**.

Domain =  $\{2, 4, 6, 8, 10, 12, 14\}$ , Range =  $\{1, 2, 3, 4, 5, 6, 7\}$ .

$\{(1, 3), (1, 5), (2, 5)\}$

Here the same first element, 1, is paired with two different second elements, 3 and 5. This violates the definition of a function, so this relation **is not a function**.

**Q2. Find the domain and range of the following real functions.**

**$f(x) = -|x|$**

The modulus  $|x|$  is defined for every real number, so the domain of  $f$  is  $\mathbb{R}$ . Since  $|x| \geq 0$  for all  $x$ , we have  $-|x| \leq 0$ , so  $f(x)$  can never be positive but can take any value from 0 downward.

Range =  $(-\infty, 0]$

**$f(x) = \sqrt{9 - x^2}$**

For the square root to be a real number, we need  $9 - x^2 \geq 0$ , i.e.  $x^2 \leq 9$ , i.e.  $-3 \leq x \leq 3$ . So Domain =  $[-3, 3]$ . As  $x$  ranges over  $[-3, 3]$ ,  $9 - x^2$  ranges from 0 (at  $x = \pm 3$ ) up to 9 (at  $x = 0$ ), so  $\sqrt{9 - x^2}$  ranges from 0 up to 3.

Range =  $[0, 3]$

**Q3. A function  $f$  is defined by  $f(x) = 2x - 5$ . Write down the values of (i)  $f(0)$ , (ii)  $f(7)$ , (iii)  $f(-3)$**

Substitute each value of  $x$  directly into  $2x - 5$ :

(i)  $f(0) = 2(0) - 5 = -5$

(ii)  $f(7) = 2(7) - 5 = 14 - 5 = 9$

(iii)  $f(-3) = 2(-3) - 5 = -6 - 5 = -11$

**Q4. The function  $t$ , which maps temperature in degree Celsius into temperature in degree Fahrenheit, is defined by  $t(C) = 9C/5 + 32$ . Find (i)  $t(0)$  (ii)  $t(28)$  (iii)  $t(-10)$  (iv) the value of  $C$ , when  $t(C) = 212$ .**

Substitute each value of  $C$  into  $t(C) = 9C/5 + 32$ :

(i)  $t(0) = 9(0)/5 + 32 = 0 + 32 = \mathbf{32}$

(ii)  $t(28) = 9(28)/5 + 32 = 252/5 + 32 = 50.4 + 32 = \mathbf{82.4}$

(iii)  $t(-10) = 9(-10)/5 + 32 = -18 + 32 = \mathbf{14}$

(iv) To find  $C$  when  $t(C) = 212$ , substitute and solve:  $212 = 9C/5 + 32$ . Subtract 32 from both sides:  $180 = 9C/5$ . Multiply both sides by 5:  $900 = 9C$ . Divide by 9:  $C = \mathbf{100}$ .

**Q5. Find the range of each of the following functions.**

**$f(x) = 2 - 3x, x \in \mathbb{R}, x > 0$**

Since  $x > 0$ , multiplying by  $-3$  (which reverses the inequality) gives  $-3x$  Range =  $(-\infty, 2)$

**$f(x) = x^2 + 2, x \text{ is a real number}$**

For every real  $x$ ,  $x^2 \geq 0$ , so adding 2 to both sides gives  $x^2 + 2 \geq 2$ . As  $x$  ranges over all reals,  $x^2$  takes every value in  $[0, \infty)$ , so  $x^2 + 2$  takes every value in  $[2, \infty)$ .

Range =  $[2, \infty)$

**$f(x) = x, x \text{ is a real number}$**

This is the identity function; as  $x$  ranges over all real numbers,  $f(x)$  does too.

Range = R

## Miscellaneous Exercise

Q1. The relation  $f$  is defined by  $f(x) = x^2$  for  $0 \leq x$

For  $f$ : The only place the two pieces could possibly clash is at the shared boundary point,  $x = 3$  (the first rule covers  $x$  up to but not including 3, and the second rule covers  $x$  from 3 onward). At  $x = 3$ , the second rule gives  $f(3) = 3 \times 3 = 9$ . Since the first rule does not apply at  $x = 3$  itself (it only applies for  $x < 3$ ),  $f$  is a function.

For  $g$ : Here both pieces are defined to include the shared boundary point  $x = 2$  ( $0 \leq x \leq 2$  for the first rule, and  $2 \leq x \leq 10$  for the second). At  $x = 2$ , the first rule gives  $g(2) = 2^2 = 4$ , while the second rule gives  $g(2) = 3 \times 2 = 6$ . The single input  $x = 2$  is thus assigned two different images, 4 and 6, which is not allowed for a function. Hence  $g$  is not a function.

Q2. If  $f(x) = x^2$ , find  $[f(1.1) - f(1)] / (1.1 - 1)$ .

Compute each piece:  $f(1.1) = (1.1)^2 = 1.21$ , and  $f(1) = (1)^2 = 1$ . So the numerator is  $f(1.1) - f(1) = 1.21 - 1 = 0.21$ , and the denominator is  $1.1 - 1 = 0.1$ .

$$[f(1.1) - f(1)] / (1.1 - 1) = 0.21 / 0.1 = 2.1$$

Q3. Find the domain of the function  $f(x) = (x^2 + 2x + 1) / (x^2 - 8x + 12)$ .

A rational function is undefined wherever its denominator equals zero, so we factorise the denominator:  $x^2 - 8x + 12 = (x - 2)(x - 6)$ , which is zero when  $x = 2$  or  $x = 6$ . The numerator does not affect the domain here since it does not create any further restriction.

So  $f$  is defined for every real number except  $x = 2$  and  $x = 6$ .

Domain of  $f = \mathbb{R} - \{2, 6\}$

Q4. Find the domain and the range of the real function  $f$  defined by  $f(x) = \sqrt{x - 1}$ .

The square root is a real number only when the quantity inside it is non-negative:  $x - 1 \geq 0$ , i.e.  $x \geq 1$ .

Domain of  $f = [1, \infty)$

Since  $x \geq 1$  means  $x - 1 \geq 0$ , the square root  $\sqrt{x - 1}$  is always  $\geq 0$ , and as  $x$  increases from 1 to  $\infty$ ,  $x - 1$  (and hence its square root) increases from 0 to  $\infty$  without bound.

Range of  $f = [0, \infty)$

**Q5.** Find the domain and the range of the real function  $f$  defined by  $f(x) = |x - 1|$ .

The expression  $|x - 1|$  is defined for every real number  $x$  (there is no restriction like a denominator or a square root here), so Domain of  $f = \mathbb{R}$ .

As  $x$  ranges over all real numbers,  $x - 1$  also ranges over all real numbers, and taking the absolute value of “all real numbers” produces every non-negative real number (and only non-negative values, since a modulus can never be negative).

Range of  $f = [0, \infty)$ , i.e. the set of all non-negative real numbers.

**Q6.** Let  $f = \{(x, x^2/(1 + x^2)) : x \in \mathbb{R}\}$  be a function from  $\mathbb{R}$  into  $\mathbb{R}$ . Determine the range of  $f$ .

Let  $y = x^2/(1 + x^2)$ . Since  $x^2 \geq 0$  for every real  $x$ , the numerator is never negative and the denominator  $1 + x^2$  is always positive, so  $y \geq 0$  for every  $x$ .

To find the upper bound, rewrite  $y$  as  $y = x^2/(1 + x^2) = [(1 + x^2) - 1]/(1 + x^2) = 1 - 1/(1 + x^2)$ . Since  $1 + x^2 \geq 1$  for all real  $x$ , the fraction  $1/(1 + x^2)$  lies in the interval  $(0, 1]$ , being closest to 1 only when  $x = 0$  and approaching 0 as  $x$  grows large. So  $y = 1 - 1/(1 + x^2)$  lies in  $[0, 1)$ , reaching 0 exactly when  $x = 0$ , and getting arbitrarily close to 1 (but never equal to 1) as  $|x|$  grows large.

Range of  $f = [0, 1)$

**Q7.** Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be defined, respectively, by  $f(x) = x + 1$ ,  $g(x) = 2x - 3$ . Find  $f + g$ ,  $f - g$  and  $f/g$ .

By definition, operations on functions are performed pointwise:  $(f + g)(x) = f(x) + g(x)$ , and similarly for subtraction and division.

$$(f + g)(x) = (x + 1) + (2x - 3) = 3x - 2$$

$$(f - g)(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$$

$$(f/g)(x) = f(x)/g(x) = (x + 1)/(2x - 3), \text{ valid wherever } g(x) \neq 0, \text{ i.e. wherever } 2x - 3 \neq 0, \text{ so } x \neq 3/2.$$

$$(f/g)(x) = (x + 1)/(2x - 3), x \neq 3/2$$

**Q8.** Let  $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$  be a function from  $\mathbb{Z}$  to  $\mathbb{Z}$  defined by  $f(x) = ax + b$ , for some integers  $a, b$ . Determine  $a, b$ .

Since  $f(x) = ax + b$ , we can substitute any two known points to form two equations. Using  $(0, -1)$ :  $f(0) = a(0) + b = b$ , and since  $f(0) = -1$ , we get  $b = -1$ .

Using  $(1, 1)$ :  $f(1) = a(1) + b = a + b$ , and since  $f(1) = 1$ , we get  $a + b = 1$ . Substituting  $b = -1$ :  $a + (-1) = 1$ , so  $a = 2$ .

Check with the remaining points:  $f(2) = 2(2) - 1 = 3$  (matches  $(2, 3)$ );  $f(-1) = 2(-1) - 1 = -3$  (matches  $(-1, -3)$ ). Both check out.

**Answer:**  $a = 2$ ,  $b = -1$ .

**Q9.** Let  $R$  be a relation from  $N$  to  $N$  defined by  $R = \{(a, b): a, b \in N \text{ and } a = b^2\}$ . Are the following true? Justify your answer in each case.

$(a, a) \in R$ , for all  $a \in N$

This would require  $a = a^2$  for every natural number  $a$ . Testing  $a = 2$ : is  $2 = 2^2 = 4$ ? No. Since the statement fails for  $a = 2$ , it is not true in general.

$(a, b) \in R$  implies  $(b, a) \in R$

Take  $a = 9$ ,  $b = 3$ : since  $9 = 3^2$ ,  $(9, 3) \in R$ . For  $(b, a) = (3, 9)$  to also be in  $R$  we would need  $3 = 9^2 = 81$ , which is false. So this is not true in general.

$(a, b) \in R$ ,  $(b, c) \in R$  implies  $(a, c) \in R$

Take  $a = 16$ ,  $b = 4$ ,  $c = 2$ : since  $16 = 4^2$ ,  $(16, 4) \in R$ ; and since  $4 = 2^2$ ,  $(4, 2) \in R$ . For  $(a, c) = (16, 2)$  to be in  $R$  we would need  $16 = 2^2 = 4$ , which is false. So this is not true in general.

**Q10.** Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{1, 5, 9, 11, 15, 16\}$  and  $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ . Are the following true? Justify your answer in each case.

$f$  is a relation from  $A$  to  $B$

A relation from  $A$  to  $B$  is, by definition, any subset of  $A \times B$ . Every ordered pair in  $f$  has its first element from  $A$  and its second element from  $B$ , so  $f \subseteq A \times B$ . Hence  $f$  is a relation from  $A$  to  $B$ .

$f$  is a function from  $A$  to  $B$

For  $f$  to be a function, every element of  $A$  must have exactly one image. Here the element 2 appears twice as a first component – once paired with 9 and once with 11 – giving it two different images. Hence  $f$  is not a function from  $A$  to  $B$ .

**Q11.** Let  $f$  be the subset of  $Z \times Z$  defined by  $f = \{(ab, a + b): a, b \in Z\}$ . Is  $f$  a function from  $Z$  to  $Z$ ? Justify your answer.

To test whether  $f$  is a function, we check whether the same first element can be

produced by two different pairs  $(a, b)$  that give different values of  $a + b$ .

Take  $a = 2, b = 6$ : then  $ab = 12$  and  $a + b = 8$ , so  $(12, 8) \in f$ . Now take  $a = -2, b = -6$ : then  $ab = (-2)(-6) = 12$  and  $a + b = -8$ , so  $(12, -8) \in f$  as well.

The same first element, 12, is thus linked to two different second elements, 8 and  $-8$ . Hence  $f$  is not a function from  $\mathbb{Z}$  to  $\mathbb{Z}$ .

**Q12.** Let  $A = \{9, 10, 11, 12, 13\}$  and let  $f: A \rightarrow \mathbb{N}$  be defined by  $f(n) =$  the highest prime factor of  $n$ . Find the range of  $f$ .

Find the prime factorisation of each element of  $A$  and pick out the largest prime factor:

$9 = 3 \times 3 \rightarrow$  highest prime factor  $= 3$

$10 = 2 \times 5 \rightarrow$  highest prime factor  $= 5$

11 is itself prime  $\rightarrow$  highest prime factor  $= 11$

$12 = 2^2 \times 3 \rightarrow$  highest prime factor  $= 3$

13 is itself prime  $\rightarrow$  highest prime factor  $= 13$

So  $f(9) = 3, f(10) = 5, f(11) = 11, f(12) = 3, f(13) = 13$ . The range collects all these output values (without repetition).

Range of  $f = \{3, 5, 11, 13\}$

## Class 11 Maths Chapter 2 – Notes and Extra Questions

Along with these NCERT solutions, students preparing for school tests and the CBSE board exam can also use our Chapter 2 Extra Questions for extra practice on Cartesian products, relations, and functions, and our Revision Notes for a quick summary of every definition, formula, and standard function graph (identity, constant, polynomial, rational, modulus, signum, and greatest integer functions) covered in this chapter.