CLASS 11 MATHS NCERT SOLUTIONS

NCERT Solutions for Class 11 Maths Chapter 3: Trigonometric Functions – Free PDF Download

Trigonometric Functions is one of the most calculation-intensive chapters in Class 11 Maths, building on the trigonometric ratios studied in earlier classes and extending them to angles of any magnitude measured in both degrees and radians. As per the 2026-27 rationalised NCERT textbook, this chapter covers Exercise...

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Trigonometric Functions is one of the most calculation-intensive chapters in Class 11 Maths, building on the trigonometric ratios studied in earlier classes and extending them to angles of any magnitude measured in both degrees and radians. As per the 2026-27 rationalised NCERT textbook, this chapter covers Exercises 3.1 to 3.3 and a Miscellaneous Exercise, dealing with radian-degree conversion, signs of trigonometric functions in different quadrants, trigonometric identities, and the sum, difference, multiple and sub-multiple angle formulas that are essential for solving trigonometric equations in the next chapter. These Class 11 Maths Chapter 3 solutions are also useful as quick revision notes before exams.

Exercise 3.1

Q1. Find the radian measures corresponding to the following degree measures:

$25°$

$-47°30'$

$240°$

$520°$

We use the relation: Radian measure = $(\pi/180) \times$ Degree measure.

(i) $25° = 25 \times \pi/180 = 5\pi/36$ radian.

(ii) $-47°30' = -(47 + 30/60)° = -47.5° = -95/2$ degree.
So, $-95/2 \times \pi/180 = -19\pi/72$ radian.

(iii) $240° = 240 \times \pi/180 = 4\pi/3$ radian.

(iv) $520° = 520 \times \pi/180 = 26\pi/9$ radian.

Q2. Find the degree measures corresponding to the following radian measures (Use π = 22/7).

$11/16$

-4

$5\pi/3$

$7\pi/6$

We use the relation: Degree measure = $(180/\pi) \times$ Radian measure.

(i) $11/16 \times 180/\pi = 11/16 \times 180 \times 7/22 = 315/8$ degree $= 39.375° = 39° + 0.375 \times 60'$ = **$39°22'30''$**.

(ii) $-4 \times 180/\pi = -4 \times 180 \times 7/22 = -2520/11 = -229.09°$.

$0.09° \times 60 = 5.45'$, and $0.45' \times 60 = 27''$. So the answer is approximately **–229°5′27″**.

(iii) $5\pi/3 \times 180/\pi = 5 \times 180/3 =$ **300°**.

(iv) $7\pi/6 \times 180/\pi = 7 \times 180/6 =$ **210°**.

Q3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Number of revolutions per second = 360/60 = 6.
One complete revolution corresponds to an angle of 2π radian.
So, in one second the wheel turns through $6 \times 2\pi =$ **12π radians**.

Q4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use π = 22/7).

Using $l = r\theta$, we get $\theta = l/r = 22/100 = 0.22$ radian.
Degree measure $= 0.22 \times 180/\pi = 0.22 \times 180 \times 7/22 = 12.6° = 12° + 0.6 \times 60' =$ **12°36′**.

Q5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Diameter = 40 cm, so radius r = 20 cm. Since the chord length (20 cm) equals the radius (20 cm), the triangle formed by the two radii and the chord is equilateral. So the angle subtended at the centre is $60° = \pi/3$ radian.
Arc length $l = r\theta = 20 \times \pi/3 =$ **20π/3 cm**.

Q6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Let the radii be r_1 and r_2, and let the common arc length be l.
$\theta_1 = 60° = \pi/3$ radian, $\theta_2 = 75° = 5\pi/12$ radian.
$l = r_1\theta_1 = r_2\theta_2 \Rightarrow r_1/r_2 = \theta_2/\theta_1 = (5\pi/12) \div (\pi/3) = (5\pi/12) \times (3/\pi) = 5/4$.
So $r_1 : r_2 =$ **5 : 4**.

Q7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length

10 cm

15 cm

21 cm

Using $\theta = l/r$ with r = 75 cm:

(i) $\theta = 10/75 =$ **2/15 radian**.

(ii) $\theta = 15/75 =$ **1/5 radian**.

(iii) $\theta = 21/75 =$ **7/25 radian**.

Exercise 3.2

Find the values of other five trigonometric functions in Exercises 1 to 5.

Q1. cos x = –1/2, x lies in third quadrant.

In the third quadrant, sin x and cos x are negative, while tan x and cot x are positive.
sec x = 1/cos x = **–2.**
$\sin^2 x = 1 - \cos^2 x = 1 - 1/4 = 3/4$, so sin x = ±√3/2. Since x is in the third quadrant, sin x = **–√3/2.**
cosec x = 1/sin x = **–2/√3 = –2√3/3.**
tan x = sin x/cos x = (–√3/2)/(–1/2) = **√3.**
cot x = 1/tan x = **1/√3 = √3/3.**

Q2. sin x = 3/5, x lies in second quadrant.

In the second quadrant, cos x, tan x, cot x and sec x are negative; cosec x is positive.
$\cos^2 x = 1 - 9/25 = 16/25$, so cos x = ±4/5. Since x is in the second quadrant, cos x = **–4/5.**
cosec x = 5/3 = **5/3.**
sec x = 1/cos x = **–5/4.**
tan x = sin x/cos x = (3/5)/(–4/5) = **–3/4.**
cot x = 1/tan x = **–4/3.**

Q3. cot x = 3/4, x lies in third quadrant.

In the third quadrant, sin x and cos x are negative, tan x and cot x are positive.
tan x = 1/cot x = **4/3.**
$\text{cosec}^2 x = 1 + \cot^2 x = 1 + 9/16 = 25/16$, so cosec x = ±5/4. Since x is in the third quadrant, cosec x = **–5/4.**
sin x = 1/cosec x = **–4/5.**
$\sec^2 x = 1 + \tan^2 x = 1 + 16/9 = 25/9$, so sec x = ±5/3. Since x is in the third quadrant, sec x = **–5/3.**
cos x = 1/sec x = **–3/5.**

Q4. sec x = 13/5, x lies in fourth quadrant.

In the fourth quadrant, cos x and sec x are positive; sin x, cosec x, tan x and cot x are negative.
cos x = 1/sec x = **5/13.**
$\sin^2 x = 1 - \cos^2 x = 1 - 25/169 = 144/169$, so sin x = ±12/13. Since x is in the fourth quadrant, sin x = **–12/13.**
cosec x = 1/sin x = **–13/12.**
tan x = sin x/cos x = (–12/13)/(5/13) = **–12/5.**
cot x = 1/tan x = **–5/12.**

Q5. tan x = –5/12, x lies in second quadrant.

In the second quadrant, cos x, tan x, cot x and sec x are negative; sin x and cosec x are positive.
cot x = 1/tan x = **–12/5**.
$\sec^2 x = 1 + \tan^2 x = 1 + 25/144 = 169/144$, so sec x = ±13/12. Since x is in the second quadrant, sec x = **–13/12**.
cos x = 1/sec x = **–12/13**.
sin x = tan x × cos x = (–5/12) × (–12/13) = **5/13**.
cosec x = 1/sin x = **13/5**.

Find the values of the trigonometric functions in Exercises 6 to 10.

Q6. sin 765°

765° = 2 × 360° + 45°. Since sin repeats after every 360°,
sin 765° = sin 45° = **1/√2**.

Q7. cosec (–1410°)

Since cosec is an odd function, cosec(–1410°) = –cosec(1410°).
1410° = 3 × 360° + 330°, so cosec(1410°) = cosec(330°).
330° lies in the fourth quadrant: sin 330° = –sin 30° = –1/2, so cosec 330° = –2.
Therefore, cosec(–1410°) = –(–2) = **2**.

Q8. tan (19π/3)

19π/3 = 6π + π/3. Since tan has period π (and also 6π is a multiple of its period),
tan(19π/3) = tan(π/3) = **√3**.

Q9. sin (–11π/3)

Since sin is an odd function, sin(–11π/3) = –sin(11π/3).
11π/3 = 2π + 5π/3, so sin(11π/3) = sin(5π/3).
5π/3 lies in the fourth quadrant: sin(5π/3) = –sin(π/3) = –√3/2.
Therefore, sin(–11π/3) = –(–√3/2) = **√3/2**.

Q10. cot (–15π/4)

Since cot is an odd function, cot(–15π/4) = –cot(15π/4).
15π/4 = 3π + 3π/4. Since cot has period π, cot(15π/4) = cot(3π/4).
cot(3π/4) = cos(3π/4)/sin(3π/4) = (–1/√2)/(1/√2) = –1.
Therefore, cot(–15π/4) = –(–1) = **1**.

Exercise 3.3

Prove that:

Q1. $\sin^2(\pi/6) + \cos^2(\pi/3) - \tan^2(\pi/4) = -1/2$

$\sin(\pi/6) = 1/2$, so $\sin^2(\pi/6) = 1/4$.
$\cos(\pi/3) = 1/2$, so $\cos^2(\pi/3) = 1/4$.
$\tan(\pi/4) = 1$, so $\tan^2(\pi/4) = 1$.
L.H.S. = 1/4 + 1/4 − 1 = 1/2 − 1 = −1/2 = R.H.S. Hence proved.

Q2. $2\sin^2(\pi/6) + \text{cosec}^2(7\pi/6)\cos^2(\pi/3) = 3/2$

$2\sin^2(\pi/6) = 2 \times 1/4 = 1/2$.
$7\pi/6$ lies in the third quadrant: $\sin(7\pi/6) = -\sin(\pi/6) = -1/2$, so $\text{cosec}(7\pi/6) = -2$, and $\text{cosec}^2(7\pi/6) = 4$.
$\cos^2(\pi/3) = 1/4$, so the second term = 4 × 1/4 = 1.
L.H.S. = 1/2 + 1 = 3/2 = R.H.S. Hence proved.

Q3. $\cot^2(\pi/6) + \text{cosec}(5\pi/6) + 3\tan^2(\pi/6) = 6$

$\cot(\pi/6) = \sqrt{3}$, so $\cot^2(\pi/6) = 3$.
$\tan(\pi/6) = 1/\sqrt{3}$, so $3\tan^2(\pi/6) = 3 \times 1/3 = 1$.
$5\pi/6$ lies in the second quadrant: $\sin(5\pi/6) = \sin(\pi - \pi/6) = \sin(\pi/6) = 1/2$, so $\text{cosec}(5\pi/6) = 2$.
L.H.S. = 3 + 2 + 1 = 6 = R.H.S. Hence proved.

Q4. $2\sin^2(3\pi/4) + 2\cos^2(\pi/4) + 2\sec^2(\pi/3) = 10$

$\sin(3\pi/4) = 1/\sqrt{2}$, so $2\sin^2(3\pi/4) = 2 \times 1/2 = 1$.
$\cos(\pi/4) = 1/\sqrt{2}$, so $2\cos^2(\pi/4) = 2 \times 1/2 = 1$.
$\sec(\pi/3) = 2$, so $2\sec^2(\pi/3) = 2 \times 4 = 8$.
L.H.S. = 1 + 1 + 8 = 10 = R.H.S. Hence proved.

Q5. Find the value of:

sin 75°

tan 15°

(i) $\sin 75° = \sin(45° + 30°) = \sin45°\cos30° + \cos45°\sin30° = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) =$ $\mathbf{(\sqrt{6} + \sqrt{2})/4}$.

(ii) $\tan 15° = \tan(45° - 30°) = (\tan45° - \tan30°)/(1 + \tan45°\tan30°) = (1 - 1/\sqrt{3})/(1 + 1/\sqrt{3}) = (\sqrt{3} - 1)/(\sqrt{3} + 1)$.
Rationalising: $= (\sqrt{3} - 1)^2/[(\sqrt{3} + 1)(\sqrt{3} - 1)] = (4 - 2\sqrt{3})/2 =$ $\mathbf{2 - \sqrt{3}}$.

Prove the following:

Q6. $\cos(\pi/4 - x)\cos(\pi/4 - y) - \sin(\pi/4 - x)\sin(\pi/4 - y) = \sin(x + y)$

Using the identity $\cos A \cos B - \sin A \sin B = \cos(A + B)$, with $A = \pi/4 - x$ and $B = \pi/4 - y$:
L.H.S. = $\cos[(\pi/4 - x) + (\pi/4 - y)] = \cos(\pi/2 - x - y) = \sin(x + y)$ = R.H.S. Hence proved.

Q7. $\tan(\pi/4 + x)/\tan(\pi/4 - x) = [(1 + \tan x)/(1 - \tan x)]^2$

$\tan(\pi/4 + x) = (1 + \tan x)/(1 - \tan x)$ and $\tan(\pi/4 - x) = (1 - \tan x)/(1 + \tan x)$.
L.H.S. = $[(1 + \tan x)/(1 - \tan x)] \div [(1 - \tan x)/(1 + \tan x)] = [(1 + \tan x)/(1 - \tan x)]^2$ = R.H.S. Hence proved.

Q8. $[\cos(\pi + x)\cos(-x)] / [\sin(\pi - x)\cos(\pi/2 + x)] = \cot^2 x$

$\cos(\pi + x) = -\cos x$, $\cos(-x) = \cos x$, so the numerator = $-\cos^2 x$.
$\sin(\pi - x) = \sin x$, $\cos(\pi/2 + x) = -\sin x$, so the denominator = $-\sin^2 x$.
L.H.S. = $(-\cos^2 x)/(-\sin^2 x) = \cos^2 x/\sin^2 x = \cot^2 x$ = R.H.S. Hence proved.

Q9. $\cos(3\pi/2 + x)\cos(2\pi + x)[\cot(3\pi/2 - x) + \cot(2\pi + x)] = 1$

$\cos(3\pi/2 + x) = \sin x$, $\cos(2\pi + x) = \cos x$, $\cot(3\pi/2 - x) = \tan x$, $\cot(2\pi + x) = \cot x$.
L.H.S. = $\sin x \cdot \cos x \cdot [\tan x + \cot x] = \sin x \cos x \cdot [(\sin^2 x + \cos^2 x)/(\sin x \cos x)] = \sin x \cos x \cdot [1/(\sin x \cos x)] = 1$ = R.H.S. Hence proved.

Q10. $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x = \cos x$

Using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, with $A = (n + 2)x$ and $B = (n + 1)x$:
L.H.S. = $\cos[(n + 2)x - (n + 1)x] = \cos x$ = R.H.S. Hence proved.

Q11. $\cos(3\pi/4 + x) - \cos(3\pi/4 - x) = -\sqrt{2} \sin x$

Using $\cos C - \cos D = -2 \sin[(C + D)/2] \sin[(C - D)/2]$, with $C = 3\pi/4 + x$ and $D = 3\pi/4 - x$:
$(C + D)/2 = 3\pi/4$ and $(C - D)/2 = x$.
L.H.S. = $-2 \sin(3\pi/4) \sin x = -2 \times (1/\sqrt{2}) \times \sin x = -\sqrt{2} \sin x$ = R.H.S. Hence proved.

Q12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

We use $\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$, obtained by writing $\sin^2 A - \sin^2 B = (\sin A + \sin B)(\sin A - \sin B)$ and applying the sum-to-product formulas.
With $A = 6x$, $B = 4x$: L.H.S. = $\sin(10x)\sin(2x)$ = R.H.S. Hence proved.

Q13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Similarly, $\cos^2 A - \cos^2 B = \sin(A + B)\sin(B - A)$. With $A = 2x$, $B = 6x$:
L.H.S. = $\sin(8x)\sin(4x)$ = R.H.S. Hence proved.

Q14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

L.H.S. = $(\sin 2x + \sin 6x) + 2\sin 4x = 2\sin 4x \cos 2x + 2\sin 4x = 2\sin 4x(\cos 2x + 1)$.
Since $\cos 2x + 1 = 2\cos^2 x$,
L.H.S. = $2\sin 4x \times 2\cos^2 x = 4\cos^2 x \sin 4x$ = R.H.S. Hence proved.

Q15. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

$\sin 5x + \sin 3x = 2\sin 4x \cos x$, so L.H.S. = $\cot 4x \times 2\sin 4x \cos x = 2\cos x(\cos 4x/\sin 4x)\sin 4x = 2\cos x \cos 4x$.
$\sin 5x - \sin 3x = 2\cos 4x \sin x$, so R.H.S. = $\cot x \times 2\cos 4x \sin x = 2\cos 4x(\cos x/\sin x)\sin x = 2\cos x \cos 4x$.

Since L.H.S. = R.H.S. = 2cosxcos4x, hence proved.

Q16. (cos9x – cos5x)/(sin17x – sin3x) = –sin2x/cos10x

cos9x – cos5x = –2sin7xsin2x.
sin17x – sin3x = 2cos10xsin7x.
L.H.S. = (–2sin7xsin2x)/(2cos10xsin7x) = –sin2x/cos10x = R.H.S. Hence proved.

Q17. (sin5x + sin3x)/(cos5x + cos3x) = tan4x

sin5x + sin3x = 2sin4xcosx.
cos5x + cos3x = 2cos4xcosx.
L.H.S. = 2sin4xcosx/2cos4xcosx = sin4x/cos4x = tan4x = R.H.S. Hence proved.

Q18. (sinx – siny)/(cosx + cosy) = tan[(x – y)/2]

sinx – siny = 2cos[(x + y)/2]sin[(x – y)/2].
cosx + cosy = 2cos[(x + y)/2]cos[(x – y)/2].
L.H.S. = sin[(x – y)/2]/cos[(x – y)/2] = tan[(x – y)/2] = R.H.S. Hence proved.

Q19. (sinx + sin3x)/(cosx + cos3x) = tan2x

sinx + sin3x = 2sin2xcosx.
cosx + cos3x = 2cos2xcosx.
L.H.S. = sin2x/cos2x = tan2x = R.H.S. Hence proved.

Q20. (sinx – sin3x)/($\sin^2$x – $\cos^2$x) = 2sinx

sinx – sin3x = 2cos2xsin(–x) = –2cos2xsinx.
$\sin^2$x – $\cos^2$x = –cos2x.
L.H.S. = (–2cos2xsinx)/(–cos2x) = 2sinx = R.H.S. Hence proved.

Q21. (cos4x + cos3x + cos2x)/(sin4x + sin3x + sin2x) = cot3x

Numerator: cos4x + cos2x = 2cos3xcosx; adding cos3x gives cos3x(2cosx + 1).
Denominator: sin4x + sin2x = 2sin3xcosx; adding sin3x gives sin3x(2cosx + 1).
L.H.S. = cos3x(2cosx + 1)/sin3x(2cosx + 1) = cos3x/sin3x = cot3x = R.H.S. Hence proved.

Q22. cotx cot2x – cot2x cot3x – cot3x cotx = 1

Since 3x = x + 2x, apply cot(A + B) = (cotA cotB – 1)/(cotA + cotB) with A = x, B = 2x:
cot3x = (cotx cot2x – 1)/(cotx + cot2x)
⇒ cot3x(cotx + cot2x) = cotx cot2x – 1
⇒ cot3x cotx + cot3x cot2x = cotx cot2x – 1
⇒ cotx cot2x – cot2x cot3x – cot3x cotx = 1. Hence proved.

Q23. tan4x = [4tanx(1 – $\tan^2$x)] / [1 – 6$\tan^2$x + $\tan^4$x]

Let t = tanx. Then tan4x = tan(2 × 2x) = 2tan2x/(1 − $\tan^2$2x), where tan2x = 2t/(1 − t^2).
1 − $\tan^2$2x = [(1 − t^2)^2 − 4t^2]/(1 − t^2)^2 = (1 − 6t^2 + t^4)/(1 − t^2)^2.
tan4x = [2 × 2t/(1 − t^2)] × [(1 − t^2)^2/(1 − 6t^2 + t^4)] = 4t(1 − t^2)/(1 − 6t^2 + t^4) = 4tanx(1 − $\tan^2$x)/(1 − 6$\tan^2$x + $\tan^4$x). Hence proved.

Q24. cos4x = 1 – 8$\sin^2$x $\cos^2$x

cos4x = 1 − 2$\sin^2$2x (using cos2θ = 1 − 2$\sin^2$θ with θ = 2x).
Since sin2x = 2sinxcosx, $\sin^2$2x = 4$\sin^2$x$\cos^2$x.
cos4x = 1 − 2(4$\sin^2$x$\cos^2$x) = 1 − 8$\sin^2$x$\cos^2$x. Hence proved.

Q25. cos6x = 32$\cos^6$x – 48$\cos^4$x + 18$\cos^2$x – 1

cos6x = cos(3 × 2x) = 4$\cos^3$(2x) − 3cos(2x) (using cos3θ = 4$\cos^3$θ − 3cosθ with θ = 2x).
Let c = cosx, so cos2x = 2c^2 − 1. Expanding (2c^2 − 1)^3 = 8c^6 − 12c^4 + 6c^2 − 1:
cos6x = 4(8c^6 − 12c^4 + 6c^2 − 1) − 3(2c^2 − 1) = 32c^6 − 48c^4 + 24c^2 − 4 − 6c^2 + 3 = 32c^6 − 48c^4 + 18c^2 − 1.
So cos6x = 32$\cos^6$x − 48$\cos^4$x + 18$\cos^2$x − 1. Hence proved.

Miscellaneous Exercise

Prove that:

Q1. 2cos(π/13)cos(9π/13) + cos(3π/13) + cos(5π/13) = 0

Using 2cosAcosB = cos(A + B) + cos(A − B) with A = π/13, B = 9π/13:
2cos(π/13)cos(9π/13) = cos(10π/13) + cos(8π/13).
cos(10π/13) = cos(π − 3π/13) = −cos(3π/13), and cos(8π/13) = cos(π − 5π/13) = −cos(5π/13).
So L.H.S. = −cos(3π/13) − cos(5π/13) + cos(3π/13) + cos(5π/13) = 0 = R.H.S. Hence proved.

Q2. (sin3x + sinx)sinx + (cos3x – cosx)cosx = 0

sin3x + sinx = 2sin2xcosx, and cos3x − cosx = −2sin2xsinx.
L.H.S. = (2sin2xcosx)sinx + (−2sin2xsinx)cosx = 2sin2xsinxcosx − 2sin2xsinxcosx = 0 = R.H.S. Hence proved.

Q3. (cosx + cosy)^2 + (sinx – siny)^2 = 4$\cos^2$[(x + y)/2]

Expanding: $\cos^2$x + 2cosxcosy + $\cos^2$y + $\sin^2$x − 2sinxsiny + $\sin^2$y
= ($\cos^2$x + $\sin^2$x) + ($\cos^2$y + $\sin^2$y) + 2(cosxcosy − sinxsiny)
= 1 + 1 + 2cos(x + y) = 2[1 + cos(x + y)] = 2 × 2$\cos^2$[(x + y)/2] = 4$\cos^2$[(x + y)/2]. Hence proved.

Q4. (cosx – cosy)^2 + (sinx – siny)^2 = 4$\sin^2$[(x – y)/2]

Expanding: $\cos^2$x − 2cosxcosy + $\cos^2$y + $\sin^2$x − 2sinxsiny + $\sin^2$y
= 2 − 2(cosxcosy + sinxsiny) = 2[1 − cos(x − y)] = 2 × 2$\sin^2$[(x − y)/2] = 4$\sin^2$[(x − y)/2]. Hence proved.

Q5. sinx + sin3x + sin5x + sin7x = 4cosx cos2x sin4x

Grouping: (sinx + sin7x) + (sin3x + sin5x) = 2sin4xcos3x + 2sin4xcosx = 2sin4x(cos3x + cosx).
cos3x + cosx = 2cos2xcosx.
L.H.S. = 2sin4x × 2cos2xcosx = 4cosxcos2xsin4x = R.H.S. Hence proved.

Q6. [(sin7x + sin5x) + (sin9x + sin3x)] / [(cos7x + cos5x) + (cos9x + cos3x)] = tan6x

sin7x + sin5x = 2sin6xcosx, and sin9x + sin3x = 2sin6xcos3x, so the numerator = 2sin6x(cosx + cos3x) = 2sin6x × 2cos2xcosx = 4sin6xcos2xcosx.
cos7x + cos5x = 2cos6xcosx, and cos9x + cos3x = 2cos6xcos3x, so the denominator = 2cos6x(cosx + cos3x) = 4cos6xcos2xcosx.
Ratio = 4sin6xcos2xcosx/4cos6xcos2xcosx = sin6x/cos6x = tan6x. Hence proved.

Q7. sin3x + sin2x – sinx = 4sinx cos(x/2) cos(3x/2)

L.H.S. = (sin3x − sinx) + sin2x = 2cos2xsinx + 2sinxcosx = 2sinx(cos2x + cosx).
cos2x + cosx = 2cos(3x/2)cos(x/2).
L.H.S. = 2sinx × 2cos(3x/2)cos(x/2) = 4sinx cos(x/2)cos(3x/2) = R.H.S. Hence proved.

Find sin(x/2), cos(x/2) and tan(x/2) for the following:

Q8. tan x = –4/3, x in quadrant II

Since 90° < x < 180°, we have 45° < x/2 < 90°, so x/2 lies in the first quadrant and sin(x/2), cos(x/2), tan(x/2) are all positive.
Since x is in quadrant II with tanx = −4/3, we get sinx = 4/5 and cosx = −3/5.
sin(x/2) = √[(1 − cosx)/2] = √[(1 + 3/5)/2] = √(4/5) = **2/√5**.
cos(x/2) = √[(1 + cosx)/2] = √[(1 − 3/5)/2] = √(1/5) = **1/√5**.
tan(x/2) = sin(x/2)/cos(x/2) = **2**.

Q9. cos x = –1/3, x in quadrant III

Since 180° < x < 270°, we have 90° < x/2 < 135°, so x/2 lies in the second quadrant: sin(x/2) is positive, while cos(x/2) and tan(x/2) are negative.
sin(x/2) = √[(1 − cosx)/2] = √[(1 + 1/3)/2] = √(2/3) = **√6/3**.
cos(x/2) = −√[(1 + cosx)/2] = −√[(1 − 1/3)/2] = −√(1/3) = **–√3/3**.
tan(x/2) = sin(x/2)/cos(x/2) = (√6/3)/(−√3/3) = −√2 × ... = **–√2**.

Q10. sin x = 1/4, x in quadrant II

Since 90° < x < 180°, x/2 lies between 45° and 90° (the first quadrant), so sin(x/2), cos(x/2) and tan(x/2) are all positive.
Since x is in quadrant II, cosx is negative: $\cos^2 x$ = 1 − 1/16 = 15/16, so cosx = −√15/4.
sin(x/2) = √[(1 − cosx)/2] = √[(4 + √15)/8].
cos(x/2) = √[(1 + cosx)/2] = √[(4 − √15)/8].
tan(x/2) = √[(4 + √15)/(4 − √15)]. Rationalising inside the root by multiplying numerator and denominator by (4 + √15), the denominator becomes 16 − 15 = 1, so tan(x/2) = $\sqrt{(4 + \sqrt{15})^2}$ = **4 + √15**.

Class 11 Maths Chapter 3 – Notes and Extra Questions

Before attempting the exercises, it helps to be clear on the key definitions and formulas of this chapter:

Degree-radian conversion: Radian measure = $(\pi/180) \times$ Degree measure, and Degree measure = $(180/\pi) \times$ Radian measure. Also, if an arc of length l subtends an angle θ (in radians) at the centre of a circle of radius r, then $l = r\theta$.

Signs in quadrants: In quadrant I all six functions are positive; in quadrant II only sin and cosec are positive; in quadrant III only tan and cot are positive; in quadrant IV only cos and sec are positive (remembered by "All Sin Tan Cos").

Basic Pythagorean identities: $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \operatorname{cosec}^2 x$.

Periodicity and allied angles: $\sin(2n\pi + x) = \sin x$ and $\cos(2n\pi + x) = \cos x$ for any integer n; $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$; $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$; $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$.

Sum and difference formulas: $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$; $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$; $\tan(x \pm y) = (\tan x \pm \tan y)/(1 \mp \tan x \tan y)$.

Double and triple angle formulas: $\sin 2x = 2\sin x \cos x$; $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$; $\tan 2x = 2\tan x/(1 - \tan^2 x)$; $\sin 3x = 3\sin x - 4\sin^3 x$; $\cos 3x = 4\cos^3 x - 3\cos x$; $\tan 3x = (3\tan x - \tan^3 x)/(1 - 3\tan^2 x)$.

Sum-to-product formulas: $\sin x + \sin y = 2\sin[(x+y)/2]\cos[(x-y)/2]$; $\sin x - \sin y = 2\cos[(x+y)/2]\sin[(x-y)/2]$; $\cos x + \cos y = 2\cos[(x+y)/2]\cos[(x-y)/2]$; $\cos x - \cos y = -2\sin[(x+y)/2]\sin[(x-y)/2]$.

Product-to-sum formulas: $2\sin x \cos y = \sin(x+y) + \sin(x-y)$; $2\cos x \sin y = \sin(x+y) - \sin(x-y)$; $2\cos x \cos y = \cos(x+y) + \cos(x-y)$; $2\sin x \sin y = \cos(x-y) - \cos(x+y)$.