CLASS 11 MATHS NCERT SOLUTIONS

NCERT Solutions for Class 11 Maths Chapter 4: Complex Numbers and Quadratic Equations – Free PDF Download

Chapter 4 of Class 11 Maths, Complex Numbers and Quadratic Equations, extends the real number system to complex numbers so that equations like $x^2 + 1 = 0$ finally have a solution. It covers the algebra of complex numbers (addition, subtraction, multiplication, division), powers of i, the modulus and conjugate of a c...

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Class 11 Maths Chapter 4 – Notes and Extra Questions

Chapter 4 of Class 11 Maths, *Complex Numbers and Quadratic Equations*, extends the real number system to complex numbers so that equations like $x^2 + 1 = 0$ finally have a solution. It covers the algebra of complex numbers (addition, subtraction, multiplication, division), powers of *i*, the modulus and conjugate of a complex number, and their representation on the Argand plane. As per the 2026-27 rationalised NCERT textbook, this chapter has a single exercise, Exercise 4.1, followed by the Miscellaneous Exercise. Below are complete, step-by-step solutions to every question. These Class 11 Maths Chapter 4 solutions are also useful as quick revision notes before exams.

Exercise 4.1

Q1. $(5i)(-\frac{3}{5}i)$

$(5i)(-\frac{3}{5}i) = 5 \times (-\frac{3}{5}) \times i \times i = -3i^2$.
Since $i^2 = -1$, this becomes $-3(-1) = 3$.
Answer: 3 (i.e., $3 + i0$)

Q2. $i^9 + i^{19}$

$i^9 = i^{4\times2+1} = i$
$i^{19} = i^{4\times4+3} = i^3 = -i$
So $i^9 + i^{19} = i + (-i) = 0$.
Answer: 0

Q3. i^{-39}

Since $i^4 = 1$, we can add any multiple of 4 to the exponent without changing the value.
$i^{-39} = i^{-39+40} = i^1 = i$.
Answer: i

Q4. $3(7 + i7) + i(7 + i7)$

$= 21 + 21i + 7i + 7i^2$
$= 21 + 28i + 7(-1)$
$= 21 - 7 + 28i = 14 + 28i$.
Answer: $14 + 28i$

Q5. $(1 - i) - (-1 + i6)$

$= 1 - i + 1 - 6i = 2 - 7i$.
Answer: $2 - 7i$

Q6. $(\frac{1}{5} + i\frac{2}{5}) - (4 + i\frac{5}{2})$

Real part: $\frac{1}{5} - 4 = \frac{1}{5} - \frac{20}{5} = -\frac{19}{5}$.
Imaginary part: $\frac{2}{5} - \frac{5}{2} = \frac{4}{10} - \frac{25}{10} = -\frac{21}{10}$.
Answer: $-\frac{19}{5} - i\frac{21}{10}$

Q7. $\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$

First combine the bracket: $(\frac{1}{3} + 4) + i(\frac{7}{3} + \frac{1}{3}) = \frac{13}{3} + i\frac{8}{3}$.
Now subtract $(-\frac{4}{3} + i)$:
Real: $\frac{13}{3} - (-\frac{4}{3}) = \frac{13}{3} + \frac{4}{3} = \frac{17}{3}$.
Imaginary: $\frac{8}{3} - 1 = \frac{5}{3}$.
Answer: $\frac{17}{3} + i\frac{5}{3}$

Q8. $(1 - i)^4$

First find $(1 - i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$.
Then $(1 - i)^4 = [(1 - i)^2]^2 = (-2i)^2 = 4i^2 = -4$.
Answer: −4

Q9. $(\frac{1}{3} + 3i)^3$

Using $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ with $x = \frac{1}{3}$, $y = 3i$:
$x^3 = \frac{1}{27}$
$3x^2y = 3 \times \frac{1}{9} \times 3i = i$
$3xy^2 = 3 \times \frac{1}{3} \times (3i)^2 = 1 \times (9i^2) = 1 \times (-9) = -9$
$y^3 = (3i)^3 = 27i^3 = 27(-i) = -27i$
Sum: $(\frac{1}{27} - 9) + i(1 - 27) = (\frac{1}{27} - \frac{243}{27}) - 26i = -\frac{242}{27} - 26i$.
Answer: $-\frac{242}{27} - 26i$

Q10. $(-2 - \frac{1}{3}i)^3$

Using $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ with $x = -2$, $y = -\frac{1}{3}i$:
$x^3 = -8$
$3x^2y = 3 \times 4 \times (-\frac{1}{3}i) = -4i$
$3xy^2 = 3 \times (-2) \times (-\frac{1}{3}i)^2 = -6 \times (-\frac{1}{9}) = \frac{2}{3}$ [since $(-\frac{1}{3}i)^2 = \frac{1}{9}i^2 = -\frac{1}{9}$]
$y^3 = (-\frac{1}{3}i)^3 = -\frac{1}{27}i^3 = -\frac{1}{27}(-i) = \frac{1}{27}i$
Sum: real part $= -8 + \frac{2}{3} = -\frac{24}{3} + \frac{2}{3} = -\frac{22}{3}$.
Imaginary part $= -4 + \frac{1}{27} = -\frac{108}{27} + \frac{1}{27} = -\frac{107}{27}$.
Answer: $-\frac{22}{3} - \frac{107}{27}i$

Find the multiplicative inverse of each of the complex numbers given in Q11 to Q13.

Q11. $4 - 3i$

Let $z = 4 - 3i$. Then $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$.
Multiplicative inverse: $z^{-1} = \bar{z}/|z|^2 = (4 + 3i)/25 = \frac{4}{25} + \frac{3}{25}i$.
Answer: $\frac{4}{25} + \frac{3}{25}i$

Q12. $\sqrt{5} + 3i$

Let $z = \sqrt{5} + 3i$. Then $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$.
$z^{-1} = (\sqrt{5} - 3i)/14 = \frac{\sqrt{5}}{14} - \frac{3}{14}i$.
Answer: $\frac{\sqrt{5}}{14} - \frac{3}{14}i$

Q13. $-i$

Let $z = -i$. Then $\bar{z} = i$ and $|z|^2 = 0^2 + (-1)^2 = 1$.
$z^{-1} = i / 1 = i$.
Check: $(-i)(i) = -i^2 = 1$ ✓
Answer: i

Q14. Express the following expression in the form a + ib: $[(3 + i\sqrt{5})(3 - i\sqrt{5})] / [(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})]$

Numerator: $(3 + i\sqrt{5})(3 - i\sqrt{5}) = 3^2 - (i\sqrt{5})^2 = 9 - (-5) = 9 + 5 = 14$.
Denominator: $(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2}) = \sqrt{3} - \sqrt{3} + \sqrt{2}i + \sqrt{2}i = 2\sqrt{2}i$.
So the expression $= 14 / (2\sqrt{2}i) = 7 / (\sqrt{2}i)$.
Multiply numerator and denominator by $-i$ (since $1/i = -i$):
$= 7 \times (-i) / \sqrt{2} = -\frac{7}{\sqrt{2}}i = -\frac{7\sqrt{2}}{2}i$ (rationalising $\frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$).
Answer: $0 - \frac{7\sqrt{2}}{2}i$

Miscellaneous Exercise

Q1. Evaluate: $[i^{18} + (\frac{1}{i})^{25}]^3$

$i^{18} = i^{4\times4+2} = i^2 = -1$.
Since $\frac{1}{i} = -i$, we get $(\frac{1}{i})^{25} = (-i)^{25} = -i^{25} = -i^{4\times6+1} = -i$.
So the bracket $= -1 + (-i) = -1 - i = -(1 + i)$.
Now $(1 + i)^2 = 1 + 2i + i^2 = 2i$, so $(1 + i)^3 = (1 + i)(2i) = 2i + 2i^2 = -2 + 2i$.
Therefore $[-(1 + i)]^3 = -(1 + i)^3 = -(-2 + 2i) = 2 - 2i$.
Answer: $2 - 2i$

Q2. For any two complex numbers z_1 and z_2, prove that $\mathrm{Re}(z_1z_2) = \mathrm{Re}\, z_1 \,\mathrm{Re}\, z_2 - \mathrm{Im}\, z_1 \,\mathrm{Im}\, z_2$.

Let $z_1 = a + i\,b$ and $z_2 = c + i\,d$, so $\mathrm{Re}\, z_1 = a$, $\mathrm{Im}\, z_1 = b$, $\mathrm{Re}\, z_2 = c$, $\mathrm{Im}\, z_2 = d$.
$z_1z_2 = (a + ib)(c + id) = ac + iad + ibc + i^2bd = (ac - bd) + i(ad + bc)$.
So $\mathrm{Re}(z_1z_2) = ac - bd = \mathrm{Re}\, z_1 \,\mathrm{Re}\, z_2 - \mathrm{Im}\, z_1 \,\mathrm{Im}\, z_2$. Hence proved.

Q3. Reduce $(\frac{1}{1-4i} - \frac{2}{1+i})(\frac{3-4i}{5+i})$ to the standard form.

First, $\frac{1}{1-4i} = \frac{1+4i}{(1-4i)(1+4i)} = \frac{1+4i}{17}$.
Also, $\frac{2}{1+i} = \frac{2(1-i)}{(1+i)(1-i)} = \frac{2(1-i)}{2} = 1 - i$.
So $\frac{1}{1-4i} - \frac{2}{1+i} = \frac{1+4i}{17} - (1 - i) = \frac{1+4i - 17(1-i)}{17} = \frac{-16+21i}{17}$.
Next, simplify $\frac{3-4i}{5+i} = \frac{(3-4i)(5-i)}{(5+i)(5-i)} = \frac{15-3i-20i+4i^2}{26} = \frac{15-23i-4}{26} = \frac{11-23i}{26}$.
Now multiply: $\frac{-16+21i}{17} \times \frac{11-23i}{26}$.
Numerator: $(-16+21i)(11-23i) = -176 + 368i + 231i - 483i^2 = -176 + 599i + 483 = 307 + 599i$.
Denominator: $17 \times 26 = 442$.
Answer: $\frac{307}{442} + \frac{599}{442}i$

Q4. If $x - iy = \sqrt{[(a-ib)/(c-id)]}$, prove that $(x^2+y^2)^2 = (a^2+b^2)/(c^2+d^2)$.

Squaring the given relation: $(x - iy)^2 = \frac{a-ib}{c-id}$ ...(1)
Taking the complex conjugate of both sides of (1): the conjugate of $(x - iy)^2$ is $(x + iy)^2$, and the conjugate of $\frac{a-ib}{c-id}$ is $\frac{a+ib}{c+id}$. So:
$(x + iy)^2 = \frac{a+ib}{c+id}$ ...(2)
Multiplying (1) and (2):
$(x - iy)^2(x + iy)^2 = \frac{(a-ib)(a+ib)}{(c-id)(c+id)}$
$[(x - iy)(x + iy)]^2 = \frac{a^2+b^2}{c^2+d^2}$
Since $(x - iy)(x + iy) = x^2 + y^2$, we get $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$. Hence proved.

Q5. If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$.

$z_1 + z_2 + 1 = (2 - i) + (1 + i) + 1 = 4$.
$z_1 - z_2 + 1 = (2 - i) - (1 + i) + 1 = 2 - 2i$.
So the ratio $= \frac{4}{2-2i} = \frac{4}{2(1-i)} = \frac{2}{1-i} = \frac{2(1+i)}{(1-i)(1+i)} = \frac{2(1+i)}{2} = 1 + i$.
$|1 + i| = \sqrt{(1^2+1^2)} = \sqrt{2}$.
Answer: $\sqrt{2}$

Q6. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Taking modulus of both sides: $|a + ib| = \frac{|(x+i)^2|}{|2x^2+1|} = \frac{|x+i|^2}{2x^2+1}$ (since $2x^2+1$ is real and positive, and $|z^2| = |z|^2$).
Squaring both sides: $|a + ib|^2 = \frac{|x+i|\text{&sup4;}}{(2x^2+1)^2}$.
Now $a^2 + b^2 = |a + ib|^2$, and $|x + i|^2 = x^2 + 1$, so |x + i|&sup4; = $(x^2+1)^2$.
Therefore $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$. Hence proved.

Q7. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find:

$\mathrm{Re}\left(\frac{z_1 z_2}{\bar{z_1}}\right)$

$\mathrm{Im}\left(\frac{1}{z_1\bar{z_1}}\right)$

(i) $z_1z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i + 1 = -3 + 4i$.
$\bar{z_1} = 2 + i$.
$\frac{z_1z_2}{\bar{z_1}} = \frac{-3+4i}{2+i} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{5} = \frac{-6+11i+4}{5} = \frac{-2+11i}{5}$.
$\mathrm{Re}\left(\frac{z_1z_2}{\bar{z_1}}\right) = -\frac{2}{5}$.
Answer (i): $-\frac{2}{5}$

(ii) $z_1\bar{z_1} = (2-i)(2+i) = 4 - i^2 = 4 + 1 = 5$ (a real number, equal to $|z_1|^2$).
$\frac{1}{z_1\bar{z_1}} = \frac{1}{5}$, which is purely real.
$\mathrm{Im}(\frac{1}{5}) = 0$.
Answer (ii): 0

Q8. Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

The conjugate of $-6 - 24i$ is $-6 + 24i$.
Expand $(x - iy)(3 + 5i) = 3x + 5xi - 3yi - 5yi^2 = (3x + 5y) + i(5x - 3y)$.

Equating with $-6 + 24i$:
$3x + 5y = -6$ ...(1)
$5x - 3y = 24$ ...(2)
Multiply (1) by 3 and (2) by 5: $9x + 15y = -18$ and $25x - 15y = 120$. Adding: $34x = 102$, so $x = 3$.
Substitute in (1): $9 + 5y = -6$, so $5y = -15$, $y = -3$.
Check in (2): $5(3) - 3(-3) = 15 + 9 = 24$ ✓
Answer: $x = 3, y = -3$

Q9. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

$\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+2i+i^2}{2} = \frac{2i}{2} = i$.
$\frac{1-i}{1+i} = \frac{(1-i)^2}{(1+i)(1-i)} = \frac{1-2i+i^2}{2} = \frac{-2i}{2} = -i$.
Difference: $i - (-i) = 2i$.
$|2i| = 2$.
Answer: 2

Q10. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

Expand: $(x + iy)^3 = x^3 + 3x^2(iy) + 3x(iy)^2 + (iy)^3$
$= x^3 + 3ix^2y - 3xy^2 - iy^3$
$= (x^3 - 3xy^2) + i(3x^2y - y^3)$.
So $u = x^3 - 3xy^2 = x(x^2 - 3y^2)$ and $v = 3x^2y - y^3 = y(3x^2 - y^2)$.
Then $\frac{u}{x} = x^2 - 3y^2$ and $\frac{v}{y} = 3x^2 - y^2$ (for $x \neq 0, y \neq 0$).
Adding: $\frac{u}{x} + \frac{v}{y} = (x^2 - 3y^2) + (3x^2 - y^2) = 4x^2 - 4y^2 = 4(x^2 - y^2)$. Hence proved.

Q11. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

We compute $|\beta-\alpha|^2$ and $|1-\bar{\alpha}\beta|^2$ and compare them.
$|\beta-\alpha|^2 = (\beta-\alpha)(\bar{\beta}-\bar{\alpha}) = |\beta|^2 - \beta\bar{\alpha} - \alpha\bar{\beta} + |\alpha|^2 = 1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2$ (using $|\beta|^2=1$, and $\beta\bar{\alpha} = \bar{\alpha}\beta$).
$|1-\bar{\alpha}\beta|^2 = (1-\bar{\alpha}\beta)(1-\alpha\bar{\beta}) = 1 - \alpha\bar{\beta} - \bar{\alpha}\beta + \bar{\alpha}\alpha\beta\bar{\beta} = 1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2|\beta|^2 = 1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2$ (again using $|\beta|^2=1$).
Both expressions are identical, so $|\beta-\alpha|^2 = |1-\bar{\alpha}\beta|^2$, which gives $|\beta-\alpha| = |1-\bar{\alpha}\beta|$.
Answer: $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

Q12. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.

$|1 - i| = \sqrt{(1^2+(-1)^2)} = \sqrt{2}$.
So the equation becomes $(\sqrt{2})^x = 2^x$, i.e., $2^{x/2} = 2^x$.
Equating exponents (same base): $\frac{x}{2} = x$, which gives $x = 0$.
Since the question asks for *non-zero* integral solutions and $x = 0$ is the only solution (and it is zero), there is no non-zero integral value of x satisfying the equation.
Answer: 0 (no non-zero integral solutions exist)

Q13. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$.

Taking the modulus of both sides of the given equation and using the property $|z_1z_2z_3z_4| = |z_1||z_2||z_3||z_4|$:

$|a+ib| \cdot |c+id| \cdot |e+if| \cdot |g+ih| = |A+iB|$.
Squaring both sides:
$|a+ib|^2 \cdot |c+id|^2 \cdot |e+if|^2 \cdot |g+ih|^2 = |A+iB|^2$
$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$. Hence proved.

Q14. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

First simplify $\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+2i+i^2}{2} = \frac{2i}{2} = i$.
So the equation becomes $i^m = 1$.
Since $i^4 = 1$ and 4 is the smallest positive integer for which this holds, the least positive integral value of m is 4.
Answer: m = 4

Class 11 Maths Chapter 4 – Notes and Extra Questions

Before attempting the exercises, it helps to be clear on the key definitions and formulas of this chapter:

Imaginary unit: $i = \sqrt{-1}$, so $i^2 = -1$. A complex number is any number of the form $a + ib$, where a and b are real numbers; a is called the real part and b the imaginary part.

Algebra of complex numbers: for $z_1 = a + ib$ and $z_2 = c + id$:

- Addition: $z_1 + z_2 = (a+c) + i(b+d)$
- Multiplication: $z_1z_2 = (ac-bd) + i(ad+bc)$
- Division ($z_2 \neq 0$): $\frac{z_1}{z_2}$ is found by multiplying numerator and denominator by the conjugate of z_2

Powers of i: $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, and the cycle repeats. In general, for any integer k: $i^{4k}=1$, $i^{4k+1}=i$, $i^{4k+2}=-1$, $i^{4k+3}=-i$. Also $\frac{1}{i} = -i$.

Modulus: for $z = a + ib$, $|z| = \sqrt{(a^2+b^2)}$, the distance of the point (a, b) from the origin on the Argand plane.

Conjugate: the conjugate of $z = a + ib$ is $\bar{z} = a - ib$. Useful identities: $z + \bar{z} = 2a$ (real), $z \cdot \bar{z} = a^2+b^2 = |z|^2$ (real and non-negative).

Multiplicative inverse: for a non-zero $z = a + ib$, $z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$.

Argand plane: a complex number $a + ib$ is plotted as the point (a, b), with the horizontal axis as the real axis and the vertical axis as the imaginary axis. The modulus |z| is the distance of this point from the origin.

Handy algebraic shortcuts: $(a+ib)(a-ib) = a^2+b^2$; $|z_1z_2| = |z_1||z_2|$; $\text{Re}(z_1z_2) = \text{Re } z_1 \text{ Re } z_2 - \text{Im } z_1 \text{ Im } z_2$. These are frequently used to simplify or prove results quickly.