CLASS 11 MATHS NCERT SOLUTIONS

NCERT Solutions for Class 11 Maths Chapter 8: Sequences and Series – Free PDF Download

Chapter 8, Sequences and Series, is one of the most important Algebra chapters in the CBSE Class 11 Maths syllabus. It builds the foundation for Arithmetic Progression (A.P.), Geometric Progression (G.P.), Geometric Mean (G.M.) and the relationship between A.M. and G.M. — concepts that reappear throughout Class 12...

Contents

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Exercise 8.1

Q1. Write the first five terms of the sequence and obtain the corresponding series: $a_n = n(n + 2)$

Substituting n = 1, 2, 3, 4, 5: $a_1 = 1(3) = 3$, $a_2 = 2(4) = 8$, $a_3 = 3(5) = 15$, $a_4 = 4(6) = 24$, $a_5 = 5(7) = 35$. The first five terms are 3, 8, 15, 24, 35 and the corresponding series is 3 + 8 + 15 + 24 + 35 + …

Q2. Write the first five terms of the sequence and obtain the corresponding series: $a_n = n \div (n + 1)$

$a_1 = 1/2$, $a_2 = 2/3$, $a_3 = 3/4$, $a_4 = 4/5$, $a_5 = 5/6$. The series is 1/2 + 2/3 + 3/4 + 4/5 + 5/6 + …

Q3. Write the first five terms of the sequence and obtain the corresponding series: $a_n = 2^n$

$a_1 = 2$, $a_2 = 4$, $a_3 = 8$, $a_4 = 16$, $a_5 = 32$. The series is 2 + 4 + 8 + 16 + 32 + …

Q4. Write the first five terms of the sequence and obtain the corresponding series: $a_n = (2n – 3)/6$

$a_1 = (2–3)/6 = –1/6$, $a_2 = (4–3)/6 = 1/6$, $a_3 = (6–3)/6 = 1/2$, $a_4 = (8–3)/6 = 5/6$, $a_5 = (10–3)/6 = 7/6$. The series is –1/6 + 1/6 + 1/2 + 5/6 + 7/6 + …

Q5. Write the first five terms of the sequence and obtain the corresponding series: $a_n = (–1)^{n–1} 5^{n+1}$

$a_1 = (–1)^0 5^2 = 25$, $a_2 = (–1)^1 5^3 = –125$, $a_3 = 5^4 = 625$, $a_4 = –5^5 = –3125$, $a_5 = 5^6 = 15625$. The series is 25 – 125 + 625 – 3125 + 15625 – …

Q6. Write the first five terms of the sequence and obtain the corresponding series: $a_n = n(n^2 + 5)/4$

$a_1 = 1(6)/4 = 3/2$, $a_2 = 2(9)/4 = 9/2$, $a_3 = 3(14)/4 = 21/2$, $a_4 = 4(21)/4 = 21$, $a_5 = 5(30)/4 = 75/2$. The series is 3/2 + 9/2 + 21/2 + 21 + 75/2 + …

Q7. Find the 17th and the 24th terms of the sequence whose nth term is $a_n = 4n – 3$.

$a_{17} = 4(17) – 3 = 68 – 3 = 65$. $a_{24} = 4(24) – 3 = 96 – 3 = 93$.

Q8. Find the 7th term of the sequence whose nth term is $a_n = n^2/2^n$.

$a_7 = 7^2/2^7 = 49/128$.

Q9. Find the 9th term of the sequence whose nth term is $a_n = (-1)^{n-1} n^3$.

$a_9 = (-1)^8(9)^3 = 1 \times 729 = 729$.

Q10. Find the 20th term of the sequence whose nth term is $a_n = n(n - 2)/(n + 3)$.

$a_{20} = 20(18)/23 = 360/23$.

Q11. Write the first five terms of the sequence defined by $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for $n > 1$, and write the corresponding series.

$a_1 = 3$. $a_2 = 3(3)+2 = 11$. $a_3 = 3(11)+2 = 35$. $a_4 = 3(35)+2 = 107$. $a_5 = 3(107)+2 = 323$. The terms are 3, 11, 35, 107, 323 and the series is 3 + 11 + 35 + 107 + 323 + …

Q12. Write the first five terms of the sequence defined by $a_1 = -1$, $a_n = a_{n-1}/n$ for $n \geq 2$.

$a_1 = -1$. $a_2 = -1/2$. $a_3 = (-1/2)/3 = -1/6$. $a_4 = (-1/6)/4 = -1/24$. $a_5 = (-1/24)/5 = -1/120$.

Q13. Write the first five terms of the sequence defined by $a_1 = a_2 = 2$, $a_n = a_{n-1} - 1$ for $n > 2$.

$a_1 = 2$, $a_2 = 2$. $a_3 = a_2 - 1 = 1$. $a_4 = a_3 - 1 = 0$. $a_5 = a_4 - 1 = -1$. The first five terms are 2, 2, 1, 0, −1.

Q14. The Fibonacci sequence is defined by $a_1 = a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$, $n > 2$. Find a_{n+1}/a_n for n = 1, 2, 3, 4, 5.

The Fibonacci terms are 1, 1, 2, 3, 5, 8, … So: for n = 1, $a_2/a_1 = 1/1 = 1$; for n = 2, $a_3/a_2 = 2/1 = 2$; for n = 3, $a_4/a_3 = 3/2$; for n = 4, $a_5/a_4 = 5/3$; for n = 5, $a_6/a_5 = 8/5$.

Exercise 8.2

Q1. Find the 20th and the nth terms of the G.P. 5/2, 5/4, 5/8, …

Here a = 5/2 and r = (5/4) ÷ (5/2) = 1/2. The nth term is $a_n = ar^{n-1} = (5/2)(1/2)^{n-1} = 5/2^n$. So $a_{20} = 5/2^{20}$.

Q2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

$a_8 = ar^7 = 192$, and r = 2, so a(128) = 192, giving a = 3/2. Then $a_{12} = ar^{11} = (3/2)(2048) = 3072$.

Q3. The 5th, 8th and 11th terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$.

$a_5 = ar^4 = p$, $a_8 = ar^7 = q$, $a_{11} = ar^{10} = s$. Then $q^2 = a^2r^{14}$ and $ps = (ar^4)(ar^{10}) = a^2r^{14}$. Since both equal a^2r^{14}, $q^2 = ps$. Hence proved.

Q4. The 4th term of a G.P. is the square of its 2nd term, and the first term is −3. Determine its 7th term.

$a_4 = ar^3$ and $a_2 = ar$. Given $ar^3 = (ar)^2 = a^2r^2$, so $r = a = -3$ (dividing by ar, $a \neq 0$, $r \neq 0$). Then $a_7 = ar^6 = (-3)(-3)^6 = (-3)(729) = -2187$.

Q5. Which term of the following sequences:

2, 2√2, 4, … is 128?
Here $r = 2\sqrt{2}/2 = \sqrt{2}$, so $a_n = 2(\sqrt{2})^{n-1} = 2^{(n+1)/2} = 128 = 2^7$. So $(n+1)/2 = 7$, giving n = 13.

√3, 3, 3√3, … is 729?
$r = \sqrt{3}$, so $a_n = (\sqrt{3})^n = 729 = 3^6 = (\sqrt{3})^{12}$. So n = 12.

1/3, 1/9, 1/27, … is 1/19683?
$r = 1/3$, so $a_n = (1/3)^n = 1/19683 = 1/3^9$. So n = 9.

Q6. For what values of x, the numbers −2/7, x, −7/2 are in G.P?

For three terms in G.P., $x^2 = (-2/7)(-7/2) = 1$. So $x = \pm 1$.

Q7. Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015, …

$a = 0.15$, $r = 0.1$. $S_{20} = a(1 - r^{20})/(1 - r) = 0.15(1 - 0.1^{20})/0.9 = (1/6)[1 - (1/10)^{20}]$.

Q8. Find the sum to n terms in the geometric progression √7, √21, 3√7, …

$a = \sqrt{7}$, $r = \sqrt{21}/\sqrt{7} = \sqrt{3}$. Since $r \neq 1$, $S_n = \sqrt{7}[(\sqrt{3})^n - 1]/(\sqrt{3} - 1)$.

Q9. Find the sum to n terms in the geometric progression 1, $-a$, a^2, $-a^3$, … (if $a \neq -1$)

Here the first term is 1 and common ratio is $-a$. So $S_n = [1 - (-a)^n]/[1 - (-a)] = [1 - (-a)^n]/(1 + a)$.

Q10. Find the sum to n terms in the geometric progression x^3, x^5, x^7, … (if $x \neq \pm 1$)

$a = x^3$, $r = x^2$. $S_n = x^3(x^{2n} - 1)/(x^2 - 1)$.

Q11. Evaluate $\sum_{k=1}^{11} (2 + 3^k)$

$\sum(2 + 3^k) = \sum 2 + \sum 3^k = 11(2) + 3(3^{11} - 1)/(3 - 1) = 22 + 3(177147 - 1)/2 = 22 + 3(177146)/2 = 22 + 265719 = 265741$.

Q12. The sum of first three terms of a G.P. is 39/10 and their product is 1. Find the common ratio and the terms.

Let the terms be a/r, a, ar. Product: $a^3 = 1$, so a = 1. Sum: $1/r + 1 + r = 39/10$, so $r + 1/r = 29/10$, i.e. $10r^2 - 29r + 10 = 0$. Solving, r = 5/2 or r = 2/5. In either case the three terms are 2/5, 1, 5/2.

Q13. How many terms of the G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

$S_n = 3(3^n - 1)/(3 - 1) = 120$, so $3^n - 1 = 80$, giving $3^n = 81 = 3^4$. Hence $n = 4$.

Q14. The sum of first three terms of a G.P. is 16, and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

First three terms: $a(1 + r + r^2) = 16$. Next three terms: $ar^3(1 + r + r^2) = 128$. Dividing, $r^3 = 8$, so $r = 2$. Substituting back, $a(7) = 16$, so $a = 16/7$. Then $S_n = a(r^n - 1)/(r - 1) = (16/7)(2^n - 1)$.

Q15. Given a G.P. with a = 729 and the 7th term equal to 64, determine S_7.

$a_7 = ar^6 = 64$, so $729r^6 = 64$, giving $r^6 = 64/729 = (2/3)^6$, so $r = 2/3$. $S_7 = 729[1 - (2/3)^7]/(1 - 2/3) = 3(2187 - 128)/\ldots$ simplifying, $S_7 = 2187 - 128 = 2059$.

Q16. Find a G.P. for which the sum of the first two terms is –4 and the 5th term is 4 times the 3rd term.

$a_5 = 4a_3$ gives $ar^4 = 4ar^2$, so $r^2 = 4$, $r = \pm 2$. Also $a(1 + r) = -4$. If $r = 2$, $a = -4/3$, giving the G.P. $-4/3, -8/3, -16/3, \ldots$ If $r = -2$, $a = 4$, giving the G.P. $4, -8, 16, -32, \ldots$

Q17. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively, prove that x, y, z are in G.P.

$x = ar^3$, $y = ar^9$, $z = ar^{15}$. Then $y^2 = a^2r^{18}$ and $xz = (ar^3)(ar^{15}) = a^2r^{18}$. Since $y^2 = xz$, x, y, z are in G.P.

Q18. Find the sum to n terms of the sequence 8, 88, 888, 8888, …

Write $S_n = 8(1 + 11 + 111 + \ldots n \text{ terms}) = (8/9)[(10 - 1) + (100 - 1) + \ldots + (10^n - 1)] = (8/9)[10(10^n - 1)/9 - n]$. This simplifies to $S_n = (8/81)(10^{n+1} - 9n - 10)$.

Q19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, 1/2.

The products are 256, 128, 64, 32, 16, which form a G.P. with $a = 256$, $r = 1/2$, $n = 5$. Sum $= 256[1 - (1/2)^5]/(1 - 1/2) = 512(31/32) = 496$.

Q20. Show that the products of corresponding terms of the sequences $a, ar, ar^2, \ldots, ar^{n-1}$ and $A, AR, AR^2, \ldots, AR^{n-1}$ form a G.P, and find the common ratio.

The kth product is $(ar^{k-1})(AR^{k-1}) = aA(rR)^{k-1}$. This is a G.P. with first term aA and common ratio rR.

Q21. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the fourth by 18.

Let the terms be a, ar, ar^2, ar^3. Given $a(r^2 - 1) = 9$ and $ar(1 - r^2) = 18$, i.e. $-ar(r^2 - 1) = 18$. Dividing, $-r = 2$, so $r = -2$. Then $3a = 9$, so $a = 3$. The four numbers are $3, -6, 12, -24$.

Q22. If the pth, qth and rth terms of a G.P. are a, b and c respectively, prove that $a^{q-r} b^{r-p} c^{p-q} = 1$.

Let A be the first term and R the common ratio: $a = AR^{p-1}$, $b = AR^{q-1}$, $c = AR^{r-1}$. The exponent of A in $a^{q-r}b^{r-p}c^{p-q}$ is (q–r)+(r–p)+(p–q) = 0, and expanding the exponent of R, (p–1)(q–r)+(q–1)(r–p)+(r–1)(p–q), also simplifies to 0. Hence the whole expression equals $A^0R^0 = 1$.

Q23. If the first and the nth term of a G.P. are a and b respectively, and if P is the product of the n terms, prove that $P^2 = (ab)^n$.

$P = a \cdot ar \cdot ar^2 \dots ar^{n-1} = a^n r^{1+2+\dots+(n-1)} = a^n r^{n(n-1)/2}$. So $P^2 = a^{2n} r^{n(n-1)}$. Also $(ab)^n = a^n(ar^{n-1})^n = a^{2n}r^{n(n-1)}$. Hence $P^2 = (ab)^n$.

Q24. Show that the ratio of the sum of the first n terms of a G.P. to the sum of terms from the (n+1)th to the (2n)th term is $1/r^n$.

$S_n = a(r^n - 1)/(r - 1)$. The sum of the next n terms (starting from ar^n) is $ar^n(r^n - 1)/(r - 1) = r^nS_n$. So the required ratio is $S_n/(r^nS_n) = 1/r^n$.

Q25. If a, b, c and d are in G.P, show that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$.

Let $b = ar$, $c = ar^2$, $d = ar^3$. Then $a^2+b^2+c^2 = a^2(1+r^2+r^4)$ and $b^2+c^2+d^2 = a^2r^2(1+r^2+r^4)$, so the LHS = $a^4r^2(1+r^2+r^4)^2$. Also $ab+bc+cd = a^2r+a^2r^3+a^2r^5 = a^2r(1+r^2+r^4)$, so RHS = $a^4r^2(1+r^2+r^4)^2$. LHS = RHS. Hence proved.

Q26. Insert two numbers between 3 and 81 so that the resulting sequence is a G.P.

Let the sequence be 3, G_1, G_2, 81. Then $r^3 = 81/3 = 27$, so r = 3. $G_1 = 3(3) = 9$ and $G_2 = 9(3) = 27$. The G.P. is 3, 9, 27, 81.

Q27. Find the value of n so that $(a^{n+1}+b^{n+1})/(a^n+b^n)$ may be the geometric mean between a and b.

Set $(a^{n+1}+b^{n+1})/(a^n+b^n) = \sqrt{ab}$. Cross-multiplying and simplifying gives $a^{n+1/2}(\sqrt{a} - \sqrt{b}) = b^{n+1/2}(\sqrt{a} - \sqrt{b})$. Since $a \neq b$, dividing both sides by $(\sqrt{a} - \sqrt{b})$ gives $a^{n+1/2} = b^{n+1/2}$, i.e. $(a/b)^{n+1/2} = 1$. This forces n + 1/2 = 0, so n = –1/2.

Q28. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio (3+2√2) : (3–2√2).

Let the numbers be a and b with $a = br^2$ for some r > 0, so √(ab) = br. Given a + b = 6√(ab): $b(r^2+1) = 6br$, so $r^2 - 6r + 1 = 0$, giving r = 3 ± 2√2. Then $a/b = r^2 = (3\pm2\sqrt{2})^2$. Because (3+2√2)(3–2√2) = 9–8 = 1, $(3+2\sqrt{2})^2$ is exactly the ratio (3+2√2):(3–2√2) written in lowest terms, so a : b = (3+2√2) : (3–2√2).

Q29. If A and G are the A.M. and G.M. respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{A^2 - G^2}$.

Let the numbers be a and b. A = (a+b)/2 so a+b = 2A, and G = √(ab) so $ab = G^2$. Then a and b are the roots of $t^2 - 2At + G^2 = 0$. By the quadratic formula, $t = [2A \pm \sqrt{(4A^2 - 4G^2)}]/2 = A \pm \sqrt{(A^2 - G^2)}$.

Q30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2nd hour, the 4th hour and the nth hour?

This is a G.P. with a = 30 and r = 2. End of 2nd hour: $30(2^2) = 120$. End of 4th hour: $30(2^4) = 480$. End of nth hour: $30(2^n)$.

Q31. What will Rs 500 amount to in 10 years after its deposit in a bank which pays annual interest at 10% compounded annually?

Since the amount grows as a G.P. with common ratio (1 + 0.1), the amount after 10 years = $500(1.1)^{10}$.

Q32. If A.M. and G.M. of two positive numbers a and b are 10 and 8 respectively, find the numbers.

A = 10 gives a+b = 20. G = 8 gives ab = 64. a and b are roots of $t^2 - 20t + 64 = 0$. The discriminant is 400 – 256 = 144, so t = (20 ± 12)/2 = 16 or 4. The numbers are 16 and 4.

Miscellaneous Exercise

Q1. Show that the sum of the (m + n)th and (m – n)th terms of an A.P. is equal to twice the mth term.

Let a and d be the first term and common difference. $a_{m+n} + a_{m-n} = [a+(m+n-1)d] + [a+(m-n-1)d] = 2a + (2m-2)d = 2[a+(m-1)d] = 2a_m$. Hence proved.

Q2. If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

Let the numbers be a–d, a, a+d. Sum: 3a = 24, so a = 8. Product: $(8-d)(8)(8+d) = 440$, so $64-d^2 = 55$, giving $d^2 = 9$, d = ±3. The numbers are 5, 8, 11.

Q3. Let the sum of n, 2n, 3n terms of an A.P. be S_1, S_2 and S_3 respectively. Show that $S_3 = 3(S_2 - S_1)$.

$S_1 = (n/2)[2a+(n-1)d]$, $S_2 = (2n/2)[2a+(2n-1)d] = n[2a+(2n-1)d]$, $S_3 = (3n/2)[2a+(3n-1)d]$. Computing $S_2 - S_1 = n[2a+(2n-1)d] - (n/2)[2a+(n-1)d] = (n/2)[2a+(3n-1)d]$, so $3(S_2-S_1) = (3n/2)[2a+(3n-1)d] = S_3$. Hence proved.

Q4. Find the sum of all numbers between 200 and 400 which are divisible by 7.

The numbers are 203, 210, ..., 399, an A.P. with a = 203, d = 7, l = 399. Number of terms: 399 = 203+(n–1)7, giving n = 29. Sum = (29/2)(203+399) = 29(301) = 8729.

Q5. Find the sum of the integers from 1 to 100 that are divisible by 2 or 5.

Sum of multiples of 2 (2 to 100, n=50) = 2550. Sum of multiples of 5 (5 to 100, n=20) = 1050. Sum of multiples of 10 (10 to 100, n=10, counted twice) = 550. By inclusion-exclusion, required sum = 2550 + 1050 − 550 = 3050.

Q6. Find the sum of all two-digit numbers which, when divided by 4, yield 1 as remainder.

These numbers are 13, 17, …, 97, an A.P. with a = 13, d = 4. Number of terms: 97 = 13+(n−1)4, giving n = 22. Sum = (22/2)(13+97) = 11(110) = 1210.

Q7. If f is a function satisfying f(x + y) = f(x) f(y) for all x, y ∈ N such that f(1) = 3 and $\sum_{x=1}^{n}$ f(x) = 120, find the value of n.

Setting x = y = 1 repeatedly shows f(1), f(2), f(3), … = 3, 9, 27, … is a G.P. with a = 3, r = 3. Sum of n terms = $3(3^n-1)/(3-1) = 120$, so $3^n-1 = 80$, $3^n = 81 = 3^4$. So n = 4.

Q8. The sum of some terms of a G.P. is 315, whose first term and common ratio are 5 and 2 respectively. Find the last term and the number of terms.

$S_n = 5(2^n-1)/(2-1) = 315$, so $2^n-1 = 63$, $2^n = 64 = 2^6$, giving n = 6. The last term = $ar^{n-1} = 5(2^5) = 160$.

Q9. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio.

a = 1, so $a_3+a_5 = r^2+r^4 = 90$. Let $u = r^2$: $u^2+u-90 = 0$, giving u = 9 or u = −10. Since $u = r^2 \geq 0$, u = 9, so r = ±3.

Q10. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an A.P. Find the numbers.

Let the numbers be a, ar, ar^2 with $a(1+r+r^2) = 56$. Since a−1, ar−7, ar^2-21 are in A.P., 2(ar−7) = $(a-1)+(ar^2-21)$, which simplifies to $a(r-1)^2 = 8$. Solving these two equations together gives r = 2, a = 8 (or r = 1/2, a = 32, the same set reversed). The numbers are 8, 16, 32.

Q11. A G.P. consists of an even number of terms. If the sum of all terms is 5 times the sum of the terms occupying odd places, find its common ratio.

Let the 2n terms be a, ar, ar^2, …, ar^{2n-1}. Total sum = $a(r^{2n}-1)/(r-1)$. Sum of odd-place terms (a, ar^2, ar^4, …) = $a(r^{2n}-1)/(r^2-1)$. Setting total = 5 × odd-sum: 1/(r−1) = 5/[(r−1)(r+1)], so r+1 = 5, giving r = 4.

Q12. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, find the number of terms.

Sum of first four terms = 4a+6d = 56. With a = 11: 44+6d = 56, so d = 2. Sum of last four terms = 4a+(4n−10)d = 112: 44+(4n−10)2 = 112, giving 4n−10 = 34, so n = 11.

Q13. If (a+bx)/(a−bx) = (b+cx)/(b−cx) = (c+dx)/(c−dx), x ≠ 0, then show that a, b, c, d are in G.P.

Cross-multiplying the first equality: $(a+bx)(b-cx) = (b+cx)(a-bx)$, which simplifies to bx(a – …) reducing to $b/a = c/b$. Similarly, cross-multiplying the second equality gives $c/b = d/c$. Hence $b/a = c/b = d/c$, so a, b, c, d are in G.P.

Q14. Let S be the sum, P the product, and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

For a G.P. a, ar, ..., ar^{n-1}: $S = a(r^n-1)/(r-1)$; $P = a^n r^{n(n-1)/2}$; $R = (1/a)[(1-(1/r)^n)/(1-1/r)] = (r^n-1)/[ar^{n-1}(r-1)]$. Then $P^2R^n = a^{2n}r^{n(n-1)} \times (r^n-1)^n/[a^n r^{n(n-1)}(r-1)^n] = a^n(r^n-1)^n/(r-1)^n = [a(r^n-1)/(r-1)]^n = S^n$.

Q15. The pth, qth and rth terms of an A.P. are a, b, c respectively. Show that $(q - r)a + (r - p)b + (p - q)c = 0$.

Let t be the first term and d the common difference: $a = t+(p-1)d$, $b = t+(q-1)d$, $c = t+(r-1)d$. Subtracting pairs: $a-b = (p-q)d$ and $b-c = (q-r)d$. Substituting into $(q-r)a+(r-p)b+(p-q)c$ and simplifying using these relations shows every term cancels, giving 0. Hence proved.

Q16. If $a(1/b + 1/c)$, $b(1/c + 1/a)$, $c(1/a + 1/b)$ are in A.P., prove that a, b, c are in A.P.

Add 1 to each term: $a/b+a/c+1$, $b/c+b/a+1$, $c/a+c/b+1$ are also in A.P. (adding a constant preserves an A.P.), i.e. $(ab+ac+bc)/bc$, $(ab+bc+ac)/ac$, $(ac+bc+ab)/ab$ are in A.P. Since $ab+bc+ca$ is common, $1/bc$, $1/ac$, $1/ab$ are in A.P., which (multiplying through by abc) means a, b, c are in A.P.

Q17. If a, b, c, d are in G.P, prove that (a^n+b^n), (b^n+c^n), (c^n+d^n) are in G.P.

Since a, b, c, d are in G.P., $b^2=ac$, $c^2=bd$, $ad=bc$. Then $(b^n+c^n)^2 = b^{2n}+2b^nc^n+c^{2n} = (ac)^n+2b^nc^n+(bd)^n = a^nc^n+b^nc^n+a^nd^n+b^nd^n$ (using $ad=bc$) $= c^n(a^n+b^n)+d^n(a^n+b^n) = (a^n+b^n)(c^n+d^n)$. Hence the three terms are in G.P.

Q18. If a and b are the roots of $x^2-3x+p = 0$ and c, d are roots of $x^2-12x+q = 0$, where a, b, c, d form a G.P., prove that $(q+p) : (q-p) = 17 : 15$.

$a+b = 3$, $ab = p$; $c+d = 12$, $cd = q$. Writing $a=x$, $b=xr$, $c=xr^2$, $d=xr^3$: $x(1+r) = 3$ and $xr^2(1+r) = 12$. Dividing gives $r^2 = 4$, $r = \pm 2$. For $r = 2$, $x = 1$, giving $ab = 2$, $cd = 32$; for $r = -2$, $x = -3$, giving $ab = -18$, $cd = -288$. In both cases, $(q+p)/(q-p) = (cd+ab)/(cd-ab)$ evaluates to 17/15 — e.g. for the first case $(32+2)/(32-2) = 34/30 = 17/15$. Hence $(q+p):(q-p) = 17:15$.

Q19. The ratio of the A.M. and G.M. of two positive numbers a and b is m : n. Show that $a : b = [m + \sqrt{(m^2-n^2)}] : [m - \sqrt{(m^2-n^2)}]$.

$[(a+b)/2] \div \sqrt{(ab)} = m/n$. Squaring and using componendo-dividendo on $(a+b)^2/(4ab) = m^2/n^2$ leads to $(a+b)^2/[(a+b)^2-4ab] = m^2/(m^2-n^2)$, i.e. $(a+b)^2/(a-b)^2 = m^2/(m^2-n^2)$, so $(a+b)/(a-b) = m/\sqrt{(m^2-n^2)}$. Applying componendo-dividendo again gives $a/b = [m+\sqrt{(m^2-n^2)}]/[m-\sqrt{(m^2-n^2)}]$.

Q20. If a, b, c are in A.P.; b, c, d are in G.P.; and 1/c, 1/d, 1/e are in A.P., prove that a, c, e are in G.P.

From a, b, c in A.P.: $b = (a+c)/2$. From b, c, d in G.P.: $d = c^2/b$. From 1/c, 1/d, 1/e in A.P.: $2/d = 1/c+1/e$. Substituting $d = c^2/b$ and $b = (a+c)/2$ into this relation and simplifying algebraically yields $c^2 = ae$, which is exactly the condition for a, c, e to be in G.P.

Q21. Find the sum of the following series up to n terms:

5 + 55 + 555 + …
$S_n = 5(1+11+111+\ldots) = (5/9)[(10-1)+(100-1)+\ldots+(10^n-1)] = (5/9)[10(10^n-1)/9 - n] = (50/81)(10^n-1) - 5n/9$.

0.6 + 0.66 + 0.666 + …
$S_n = 6(0.1+0.11+0.111+\ldots) = (6/9)[(1-0.1)+(1-0.01)+\ldots] = (2/3)[n - (1-0.1^n)/9] \times \ldots$ simplified: $S_n = (2/3)n - (2/27)(1-10^{-n})$.

Q22. Find the 20th term of the series 2×4 + 4×6 + 6×8 + … + n terms.

The nth term is $a_n = (2n)(2n+2) = 4n^2+4n$. So $a_{20} = 4(400)+4(20) = 1600+80 = 1680$.

Q23. Find the sum of the first n terms of the series 3 + 7 + 13 + 21 + 31 + …

The differences 4, 6, 8, 10, … are themselves in A.P., so the nth term is $a_n = 3 + [4+6+8+\ldots(n-1) \text{ terms}] = 3 + (n-1)(n+4) = n^2+n+1$. Summing, $S_n = \sum(k^2+k+1) = n(n+1)(2n+1)/6 + n(n+1)/2 + n$.

Q24. If S_1, S_2, S_3 are the sums of the first n natural numbers, their squares and their cubes respectively, show that $9S_2^2 = S_3(1+8S_1)$.

$S_1 = n(n+1)/2$, $S_2 = n(n+1)(2n+1)/6$, $S_3 = [n(n+1)/2]^2$. Then $1+8S_1 = 1+4n(n+1) = (2n+1)^2$, so $S_3(1+8S_1) = [n(n+1)/2]^2(2n+1)^2$. Also $9S_2^2 = 9[n(n+1)(2n+1)/6]^2 = [n(n+1)(2n+1)/2]^2$, which is the same expression. Hence $9S_2^2 = S_3(1+8S_1)$.

Q25. Find the sum of the series $1^3/1 + (1^3+2^3)/(1+3) + (1^3+2^3+3^3)/(1+3+5) + \ldots$ up to n terms.

The kth term has numerator $1^3+\ldots+k^3 = [k(k+1)/2]^2$ and denominator $1+3+\ldots+(2k-1) = k^2$, so the kth term simplifies to $(k+1)^2/4$. Summing for k = 1 to n: $S_n = (1/4)\sum(k+1)^2 = (1/4)[n(n+1)(2n+1)/6 + n(n+1) + n]$, which simplifies to $S_n = n(n+1)(n+2)/6$.

Q26. Show that $1\times2^2 + 2\times3^2 + \ldots + n\times(n+1)^2$ all over $1^2\times2 + 2^2\times3 + \ldots + n^2\times(n+1)$ equals $(3n+5)/(3n+1)$.

The nth term of the numerator series is $n(n+1)^2 = n^3+2n^2+n$, and the nth term of the denominator series is $n^2(n+1) = n^3+n^2$. Summing both using S_1, S_2, S_3 formulas and simplifying the ratio of the two sums (after factoring out common terms of n(n+1)) gives exactly (3n+5)/(3n+1).

Q27. A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual installments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

Unpaid amount = Rs 6000, repaid as 12 installments of Rs 500. Interest is paid on 6000, 5500, 5000, …, 500 — an A.P. with a = 500, d = 500, n = 12, sum = (12/2)[2(500)+11(500)] = 6(6500) = 39000. Total interest = 12% of 39000 = Rs 4680. Total cost = 12000+4680 = Rs 16680.

Q28. Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual installments of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Unpaid amount = Rs 18000, in 18 installments. Interest is on 18000, 17000, ..., 1000 — an A.P. summing to (18/2)(18000+1000) = 9(19000) = 171000. Total interest = 10% of 171000 = Rs 17100. Total cost = 22000+17100 = Rs 39100.

Q29. A person writes a letter to four of his friends and asks each to copy it and mail it to four different persons, continuing the chain. Assuming the chain is unbroken and it costs 50 paise to mail one letter, find the amount spent on postage when the 8th set of letters is mailed.

The number of letters mailed at each stage forms a G.P.: 4, 4^2, ..., 4^8, with a = 4, r = 4, n = 8. Total letters = $4(4^8-1)/(4-1)$ = 87380. Cost = 87380 × Rs 0.50 = Rs 43690.

Q30. A man deposited Rs 10000 in a bank at 5% simple interest annually. Find the amount in the 15th year since he deposited the amount, and also the total amount after 20 years.

Annual simple interest = 5% of 10000 = Rs 500. Amount at start of 15th year = 10000 + 14(500) = Rs 17000. Amount after 20 years = 10000 + 20(500) = Rs 20000.

Q31. A manufacturer reckons that the value of a machine, which costs him Rs 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Each year the value becomes 4/5 of the previous value, forming a G.P. with a = 15625, r = 4/5. Value after 5 years = $15625(4/5)^5$ = 15625 × 1024/3125 = 5120. So the machine is worth Rs 5120.

Q32. 150 workers were engaged to finish a job in a certain number of days. Four workers dropped out on the second day, four more on the third day, and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

Let x be the number of days planned for 150 workers, so total work = 150x worker-days. With workers dropping by 4 each day, the actual work done over (x+8) days is an A.P. with a = 150, d = −4, n = x+8, and this sum must equal 150x. Setting up and solving (x+8)/2 × [300−4(x+7)] = 150x leads to $x^2-29x-272=0$ type simplification and gives x = 17 (rejecting the negative root). So the work was actually completed in 17+8 = 25 days.

Class 11 Maths Chapter 8 – Notes and Extra Questions

Remember these key formulas while revising: for an A.P. with first term a and common difference d, the nth term is $a_n = a + (n-1)d$ and the sum of n terms is $S_n = (n/2)[2a+(n-1)d]$. For a G.P. with first term a and common ratio r, the nth term is $a_n = ar^{n-1}$ and the sum of n terms is $S_n = a(r^n-1)/(r-1)$ for r ≠ 1 (or S_n = na for r = 1). The Geometric Mean of two positive numbers a and b is √(ab), and the A.M.–G.M. relationship states A ≥ G always, with equality only when a = b. Also keep the special-series results handy: 1+2+...+n = n(n+1)/2, $1^2+2^2+...+n^2$ = n(n+1)(2n+1)/6, and $1^3+2^3+...+n^3$ = $[n(n+1)/2]^2$ — these appear in Miscellaneous Exercise questions even though they no longer have a dedicated practice exercise of their own.

How many exercises are there in Class 11 Maths Chapter 8, Sequences and Series?

In the current (2023-rationalised) NCERT textbook there are two exercises — Exercise 8.1 (14 questions on sequences) and Exercise 8.2 (32 questions on Geometric Progression and the A.M.–G.M. relationship) — plus a Miscellaneous Exercise with 32 questions. That is 78 questions in total.

Was Arithmetic Progression removed from this chapter under the new NCERT syllabus?

Arithmetic Progression is still explained in the chapter's theory section with worked examples, and A.P.-based problems still appear throughout the Miscellaneous Exercise. However, the dedicated stand-alone practice exercise on A.P. (the old Exercise 9.2 from the pre-2023 edition) was dropped during rationalisation, since Class 10 Chapter 5 already covers Arithmetic Progressions in full depth. Similarly, the old exercise on "special series" (sums of squares and cubes, previously Exercise 9.4) was removed as a separate exercise, though its formulas are still used in Miscellaneous Exercise questions like Q24. The chapter itself was also renumbered from Chapter 9 (pre-2023 books) to Chapter 8.

What is the difference between a sequence and a series?

A sequence is an ordered list of numbers following a definite rule (each term has a fixed position), such as a_1, a_2, a_3, ... A series is what you get when you add the terms of a sequence together, such as $a_1 + a_2 + a_3 + ...$ So every series comes from a sequence, but a series represents a sum, not just a list.

What is the relationship between the Arithmetic Mean (A.M.) and Geometric Mean (G.M.) of two positive numbers?

For any two positive real numbers a and b, if A is their A.M. and G is their G.M., then A ≥ G always holds, with equality if and only if a = b. This is proved in the chapter by showing $A - G = (\sqrt{a} - \sqrt{b})^2/2 \geq 0$, and it is the basis of several proof-based questions in Exercise 8.2 and the Miscellaneous Exercise.