

CLASS 12

MATHEMATICS

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NCERT Solutions for Class 12 Mathematics Chapter 10: Vectors – Free PDF Download

Chapter 10 of Class 12 Maths is Vectors, covering vector representation, algebra, dot and cross products, and their geometric applications.

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Class 12 Mathematics Chapter 10 – Notes and Extra Questions

Chapter 10 of Class 12 Maths is Vectors, covering vector representation, algebra, dot and cross products, and their geometric applications.

Vector Basics

A **vector** has both magnitude and direction, denoted **a** or with an arrow. **Position vectors** locate points relative to the origin. Unit vectors **i**, **j**, **k** represent the coordinate axes.

Types of Vectors

Zero vector (magnitude 0), **unit vector** (magnitude 1), **collinear vectors** (parallel to the same line), **equal vectors** (same magnitude and direction), **negative vector** (same magnitude, opposite direction).

Algebra of Vectors

Addition follows the triangle/parallelogram law. **Scalar multiplication** scales magnitude, preserving or reversing direction based on sign. A vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ has magnitude $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Section Formula

The position vector of a point dividing the line joining points with position vectors **a** and **b** in ratio **m:n** is $(m\mathbf{b} + n\mathbf{a})/(m+n)$.

Dot (Scalar) Product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$. Used to find the angle between vectors and to check perpendicularity ($\mathbf{a} \cdot \mathbf{b} = 0$).

Cross (Vector) Product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta \hat{n}$ (a vector perpendicular to both), computed via a 3×3 determinant with **i**, **j**, **k**. Used to find area of a triangle/parallelogram and to check parallelism ($\mathbf{a} \times \mathbf{b} = 0$).

Exercise 10.1 Solutions (All 5 Questions)

1. Represent graphically a displacement of 40 km, 30° east of north.

Ans: Choosing a suitable scale (say 1 cm = 10 km), the displacement is represented by a line segment of length 4 cm starting from the initial point O, drawn in a direction that makes an angle of 30° with the north direction, measured towards the east. The arrowhead at the terminal point P shows the direction of the displacement, so the vector OP represents the given displacement of 40 km, 30° east of north.

2. Classify the following measures as scalars and vectors: (i) 10 kg (ii) 2 metres north-west (iii) 40° (iv) 40 watt (v) 10⁻¹⁹ coulomb (vi) 20 m/s²

Ans: (i) 10 kg is a scalar, as it involves only magnitude.

(ii) 2 metres north-west is a vector, as it involves both magnitude and direction.

(iii) 40° is a scalar, as it involves only magnitude.

(iv) 40 watt is a scalar, as it involves only magnitude.

(v) 10⁻¹⁹ coulomb is a scalar, as it involves only magnitude.

(vi) 20 m/s² is a vector, as it involves both magnitude and direction.

3. Classify the following as scalar and vector quantities: (i) time period (ii) distance (iii) force (iv) velocity (v) work done

Ans: (i) Time period is a scalar quantity, as it involves only magnitude.
 (ii) Distance is a scalar quantity, as it involves only magnitude.
 (iii) Force is a vector quantity, as it involves both magnitude and direction.
 (iv) Velocity is a vector quantity, as it involves both magnitude and direction.
 (v) Work done is a scalar quantity, as it involves only magnitude.

4. In the given figure, identify the following vectors: (i) co-initial (ii) equal (iii) collinear but not equal

Ans: In the standard figure for this question, the vectors a, b, c, d, e, and f are drawn along the sides and a diagonal of a rectangle-like arrangement.

(i) Vectors a and d are co-initial, as they both start from the same initial point.
 (ii) Vectors b and d are equal, as they have the same magnitude and the same direction.
 (iii) Vectors a and c are collinear but not equal. They are parallel (lie along the same line or parallel lines) but are not equal, since although they are parallel, their directions are not the same.

5. Answer the following as true or false: (i) a and -a are collinear (ii) two collinear vectors are always equal in magnitude (iii) two vectors having the same magnitude are collinear (iv) two collinear vectors having the same magnitude are equal

Ans: (i) True. Vectors a and -a are collinear, since they lie along the same line, just in opposite directions.
 (ii) False. Collinear vectors need only be parallel to the same line; their magnitudes may be different.
 (iii) False. Two vectors having the same magnitude need not be parallel to the same line, so they need not be collinear.
 (iv) False. Two collinear vectors of the same magnitude can point in opposite directions (like a and -a), in which case they are not equal.

Exercise 10.2 Solutions (All 19 Questions)

1. Compute the magnitude of the following vectors: $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$; $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

Ans: $|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$.
 $|\vec{b}| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}$.
 $|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$.

2. Write two different vectors having the same magnitude.

Ans: Let $\vec{p} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{q} = 2\hat{i} - \hat{j} - 3\hat{k}$. Then $|\vec{p}| = \sqrt{1+4+9} = \sqrt{14}$ and $|\vec{q}| = \sqrt{4+1+9} = \sqrt{14}$. Both vectors have the same magnitude $\sqrt{14}$ but point in different directions, so they are different (unequal) vectors.

3. Write two different vectors having the same direction.

Ans: Let $\vec{p} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{q} = 2\hat{i} + 2\hat{j} + 2\hat{k}$. The direction cosines of $\vec{p}$ are $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, and the direction cosines of $\vec{q}$ are $(\frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}}) = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ as well. Since their direction cosines are identical, $\vec{p}$ and $\vec{q}$ have the same direction, though they are different vectors (different magnitudes).

4. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Ans: Two vectors are equal only when their corresponding components are equal. Comparing the coefficients of $\hat{i}$ and $\hat{j}$, $x = 2$ and $y = 3$.

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

Ans: $\vec{PQ} = (-5-2)\hat{i} + (7-1)\hat{j} = -7\hat{i} + 6\hat{j}$. So the scalar components are -7 and 6, and the vector components

are $-7\hat{i}$ and $6\hat{j}$.

6. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Ans: $\vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} = 0\hat{i} - 4\hat{j} - \hat{k} = -4\hat{j} - \hat{k}$.

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Ans: $|\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$. So $\hat{a} = \vec{a}/|\vec{a}| = (1/\sqrt{6})\hat{i} + (1/\sqrt{6})\hat{j} + (2/\sqrt{6})\hat{k}$.

8. Find the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Ans: $\vec{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$. $|\vec{PQ}| = \sqrt{9+9+9} = 3\sqrt{3}$. So the unit vector is $\vec{PQ}/|\vec{PQ}| = (1/\sqrt{3})\hat{i} + (1/\sqrt{3})\hat{j} + (1/\sqrt{3})\hat{k}$.

9. For the given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Ans: $\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2+1)\hat{k} = \hat{i} + \hat{k}$. $|\vec{a} + \vec{b}| = \sqrt{1+0+1} = \sqrt{2}$. So the unit vector is $(1/\sqrt{2})\hat{i} + (1/\sqrt{2})\hat{k}$.

10. Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Ans: Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$. $|\vec{a}| = \sqrt{25+1+4} = \sqrt{30}$. So the unit vector is $\hat{a} = (5/\sqrt{30})\hat{i} - (1/\sqrt{30})\hat{j} + (2/\sqrt{30})\hat{k}$. Therefore, the required vector of magnitude 8 units is $8\hat{a} = (40/\sqrt{30})\hat{i} - (8/\sqrt{30})\hat{j} + (16/\sqrt{30})\hat{k}$.

11. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Ans: Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$. Then $\vec{b} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$, so $\vec{b} = \lambda\vec{a}$ with $\lambda = -2$. Since $\vec{b}$ is a scalar multiple of $\vec{a}$, the two vectors are collinear.

12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Ans: Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. $|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$. So the direction cosines are $(1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$.

13. Find the direction cosines of the vector joining the points A(1, 2, -3) and B(-1, -2, 1), directed from A to B.

Ans: $\vec{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1-(-3))\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$. $|\vec{AB}| = \sqrt{4+16+16} = 6$. So the direction cosines are $(-2/6, -4/6, 4/6) = (-1/3, -2/3, 2/3)$.

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

Ans: Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. $|\vec{a}| = \sqrt{3}$, so the direction cosines are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. Since $\cos\alpha = \cos\beta = \cos\gamma = 1/\sqrt{3}$, where α, β, γ are the angles $\vec{a}$ makes with OX, OY, OZ, the vector is equally inclined to all three axes.

15. Find the position vector of a point R which divides the line joining two points P and Q, whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2:1 (i) internally (ii) externally.

Ans: (i) Internally: $\vec{OR} = [2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})]/(2+1) = (-\hat{i} + 4\hat{j} + \hat{k})/3 = (-1/3)\hat{i} + (4/3)\hat{j} + (1/3)\hat{k}$.

(ii) Externally: $\vec{OR} = [2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})]/(2-1) = -3\hat{i} + 3\hat{k}$.

16. Find the position vector of the midpoint of the vector joining the points P(2, 3, 4) and Q(4, 1, -2).

Ans: The position vector of the midpoint R is $\vec{OR} = [(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})]/2 = (6\hat{i} + 4\hat{j} + 2\hat{k})/2 = 3\hat{i} + 2\hat{j} + \hat{k}$.

17. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right-angled triangle.

Ans: $\vec{AB} = \vec{b} - \vec{a} = -\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{BC} = \vec{c} - \vec{b} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\vec{CA} = \vec{a} - \vec{c} = 2\hat{i} - \hat{j} + \hat{k}$. Then $|\vec{AB}|^2 = 1+9+25 = 35$, $|\vec{BC}|^2 = 1+4+36 = 41$, $|\vec{CA}|^2 = 4+1+1 = 6$. Since $|\vec{AB}|^2 + |\vec{CA}|^2 = 35+6 = 41 = |\vec{BC}|^2$, the triangle is right-angled at C.

$= |\vec{BC}|^2$, the triangle satisfies the Pythagoras theorem, so ABC is a right-angled triangle (right angle at A).

18. In triangle ABC, which of the following is not true? (A) $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$ (B)

$\vec{AB} + \vec{BC} - \vec{AC} = \vec{0}$ (C) $\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$ (D) $\vec{AB} - \vec{CB} + \vec{CA} = \vec{0}$

Ans: By the triangle law of addition, $\vec{AB} + \vec{BC} = \vec{AC}$. This gives $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$ (option A is true), $\vec{AB} + \vec{BC} - \vec{AC} = \vec{0}$ (option B is true), and $\vec{AB} - \vec{CB} + \vec{CA} = \vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$ (option D is true). But $\vec{AB} + \vec{BC} - \vec{CA} = \vec{AC} - \vec{CA} = 2\vec{AC} \neq \vec{0}$ in general, so option (C) is not true. The answer is (C).

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect? (A)

$\vec{b} = \lambda\vec{a}$, for some scalar λ (B) $\vec{a} = \pm\vec{b}$ (C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional (D) both the vectors $\vec{a}$ and $\vec{b}$ have the same direction, but different magnitudes

Ans: If $\vec{a}$ and $\vec{b}$ are collinear, they are parallel, so $\vec{b} = \lambda\vec{a}$ for some scalar λ , which makes statement (A) correct. Writing $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, the relation $\vec{b} = \lambda\vec{a}$ gives $\lambda = b_1/a_1 = b_2/a_2 = b_3/a_3$, which shows that the respective components of $\vec{a}$ and $\vec{b}$ are always proportional. So the statement in (C), that they are “not proportional”, is incorrect. The answer is (C).

Exercise 10.3 Solutions (All 18 Questions)

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

Ans: Using $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, we get $\sqrt{6} = \sqrt{3} \cdot 2 \cdot \cos\theta$, so $\cos\theta = \sqrt{6}/(2\sqrt{3}) = 1/\sqrt{2}$. Hence $\theta = \pi/4$.

2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Ans: Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$. Then $\vec{a} \cdot \vec{b} = 3 + 4 + 3 = 10$, $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{14}$. So $\cos\theta = 10/14 = 5/7$, giving $\theta = \cos^{-1}(5/7)$.

3. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

Ans: Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$. The projection of $\vec{a}$ on $\vec{b}$ is $\vec{a} \cdot \vec{b} / |\vec{b}| = (1-1)/\sqrt{2} = 0$.

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

Ans: Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$. $\vec{a} \cdot \vec{b} = 7 - 3 + 56 = 60$. $|\vec{b}| = \sqrt{(49+1+64)} = \sqrt{114}$. So the projection of $\vec{a}$ on $\vec{b}$ is $60/\sqrt{114}$.

5. Show that each of the given three vectors is a unit vector: $(1/7)(2\hat{i}+3\hat{j}+6\hat{k})$, $(1/7)(3\hat{i}-6\hat{j}+2\hat{k})$, $(1/7)(6\hat{i}+2\hat{j}-3\hat{k})$. Also show that they are mutually perpendicular to each other.

Ans: Let $\vec{a} = (1/7)(2\hat{i}+3\hat{j}+6\hat{k})$, $\vec{b} = (1/7)(3\hat{i}-6\hat{j}+2\hat{k})$, $\vec{c} = (1/7)(6\hat{i}+2\hat{j}-3\hat{k})$. $|\vec{a}| = (1/7)\sqrt{(4+9+36)} = 7/7 = 1$, $|\vec{b}| = (1/7)\sqrt{(9+36+4)} = 1$, $|\vec{c}| = (1/7)\sqrt{(36+4+9)} = 1$, so all three are unit vectors. Also $\vec{a} \cdot \vec{b} = (1/49)(6-18+12) = 0$, $\vec{b} \cdot \vec{c} = (1/49)(18-12-6) = 0$, $\vec{a} \cdot \vec{c} = (1/49)(12+6-18) = 0$, so the three vectors are mutually perpendicular.

6. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

Ans: $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 8$. Substituting $|\vec{a}| = 8|\vec{b}|$, we get $64|\vec{b}|^2 - |\vec{b}|^2 = 8$, so $63|\vec{b}|^2 = 8$, giving $|\vec{b}| = \sqrt{(8/63)}$. Hence $|\vec{a}| = 8\sqrt{(8/63)}$.

7. Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$.

Ans: $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) = 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b} = 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$.

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $1/2$.

Ans: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos 60^\circ$. Since $|\vec{a}| = |\vec{b}|$, this gives $1/2 = |\vec{a}|^2 \cdot (1/2)$, so $|\vec{a}|^2 = 1$, giving $|\vec{a}| = 1$.

$$|\mathbf{b}| = 1.$$

9. Find $|\mathbf{x}|$, if for a unit vector $\mathbf{a}$, $(\mathbf{x}-\mathbf{a}) \cdot (\mathbf{x}+\mathbf{a}) = 12$.

Ans: $(\mathbf{x}-\mathbf{a}) \cdot (\mathbf{x}+\mathbf{a}) = |\mathbf{x}|^2 - |\mathbf{a}|^2 = 12$. Since $|\mathbf{a}| = 1$, $|\mathbf{x}|^2 = 13$, so $|\mathbf{x}| = \sqrt{13}$.

10. If $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$ are such that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$, then find the value of λ .

Ans: $\mathbf{a} + \lambda\mathbf{b} = (2-\lambda)\mathbf{i} + (2+2\lambda)\mathbf{j} + (3+\lambda)\mathbf{k}$. Since this is perpendicular to $\mathbf{c}$, $(\mathbf{a} + \lambda\mathbf{b}) \cdot \mathbf{c} = 0$, so $3(2-\lambda) + (2+2\lambda) = 0$, giving $6-3\lambda+2+2\lambda = 0$, so $8-\lambda = 0$, hence $\lambda = 8$.

11. Show that $|\mathbf{a}||\mathbf{b}| + |\mathbf{b}||\mathbf{a}|$ is perpendicular to $|\mathbf{a}||\mathbf{b}| - |\mathbf{b}||\mathbf{a}|$, for any two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$.

Ans: Let $\mathbf{p} = |\mathbf{a}||\mathbf{b}| + |\mathbf{b}||\mathbf{a}|$ and $\mathbf{q} = |\mathbf{a}||\mathbf{b}| - |\mathbf{b}||\mathbf{a}|$. Then $\mathbf{p} \cdot \mathbf{q} = |\mathbf{a}|^2(|\mathbf{b}|^2 - |\mathbf{b}|^2) - |\mathbf{b}|^2(|\mathbf{a}|^2 - |\mathbf{a}|^2) = 0$. Since $\mathbf{p} \cdot \mathbf{q} = 0$, the two vectors are perpendicular.

12. If $\mathbf{a} \cdot \mathbf{a} = 0$ and $\mathbf{a} \cdot \mathbf{b} = 0$, then what can be concluded about the vector $\mathbf{b}$?

Ans: Since $\mathbf{a} \cdot \mathbf{a} = 0$, $|\mathbf{a}|^2 = 0$, so $\mathbf{a}$ is the zero vector. Then $\mathbf{a} \cdot \mathbf{b} = 0$ holds automatically for the zero vector regardless of $\mathbf{b}$, so $\mathbf{b}$ can be any vector.

13. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, find the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$.

Ans: Squaring, $(\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0$, so $|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$. Since each vector is a unit vector, $1+1+1+2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$, giving $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = -3/2$.

14. If either $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$, then $\mathbf{a} \cdot \mathbf{b} = 0$. But the converse need not be true. Justify your answer with an example.

Ans: Take $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. Then $\mathbf{a} \cdot \mathbf{b} = 6 + 12 - 18 = 0$, but clearly neither $\mathbf{a} = \mathbf{0}$ nor $\mathbf{b} = \mathbf{0}$. This shows that $\mathbf{a} \cdot \mathbf{b} = 0$ does not require either vector to be the zero vector.

15. If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0) and (0,1,2) respectively, then find $\angle ABC$.

Ans: $\angle ABC$ is the angle between $\mathbf{BA}$ and $\mathbf{BC}$. $\mathbf{BA} = (2,2,3)$, $\mathbf{BC} = (1,1,2)$. $\mathbf{BA} \cdot \mathbf{BC} = 2+2+6 = 10$, $|\mathbf{BA}| = \sqrt{17}$, $|\mathbf{BC}| = \sqrt{6}$. So $\cos(\angle ABC) = 10/(\sqrt{17}\sqrt{6})$, giving $\angle ABC = \cos^{-1}(10/\sqrt{102})$.

16. Show that the points A(1,2,7), B(2,6,3) and C(3,10,-1) are collinear.

Ans: $\mathbf{AB} = \mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$, $\mathbf{BC} = \mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$, $\mathbf{AC} = 2(\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})$. So $|\mathbf{AB}| = |\mathbf{BC}| = \sqrt{33}$ and $|\mathbf{AC}| = 2\sqrt{33}$. Since $|\mathbf{AC}| = |\mathbf{AB}| + |\mathbf{BC}|$, the points A, B, C are collinear (with B the midpoint of AC).

17. Show that the vectors $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$, $3\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ form the vertices of a right-angled triangle.

Ans: Let these be position vectors of A, B, C respectively. $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$, $\mathbf{BC} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{CA} = -\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$. Then $|\mathbf{AB}|^2 = 1+4+36 = 41$, $|\mathbf{BC}|^2 = 4+1+1 = 6$, $|\mathbf{CA}|^2 = 1+9+25 = 35$. Since $|\mathbf{BC}|^2 + |\mathbf{CA}|^2 = 6+35 = 41 = |\mathbf{AB}|^2$, the triangle satisfies the Pythagoras theorem, so it is right-angled (right angle at C).

18. If $\mathbf{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\mathbf{a}$ is a unit vector for what value of λ ?

Ans: For $\lambda\mathbf{a}$ to be a unit vector, $|\lambda\mathbf{a}| = 1$, i.e. $|\lambda||\mathbf{a}| = 1$, so $|\lambda| = 1/a$.

Exercise 10.4 Solutions (All 12 Questions)

1. Find $|\mathbf{a} \times \mathbf{b}|$, if $\mathbf{a} = \mathbf{i} - 7\mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

Ans: $\mathbf{a} \times \mathbf{b} = \mathbf{i}(-14+14) - \mathbf{j}(2-21) + \mathbf{k}(-2+21) = 19\mathbf{j} + 19\mathbf{k}$. So $|\mathbf{a} \times \mathbf{b}| = \sqrt{(19^2+19^2)} = 19\sqrt{2}$.

2. Find a unit vector perpendicular to each of the vectors $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - 2\vec{k}$, where $\vec{a} = 3\vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - 2\vec{k}$.

Ans: $\vec{a} = 3\vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - 2\vec{k}$. Their cross product is $(\vec{a} \times \vec{b}) = \hat{i}(16-0) - \hat{j}(16-0) + \hat{k}(0-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$, with magnitude $\sqrt{(256+256+64)} = \sqrt{576} = 24$. So the required unit vector is $\pm(16\hat{i} - 16\hat{j} - 8\hat{k})/24 = \pm(2/3)\hat{i} \mp (2/3)\hat{j} \mp (1/3)\hat{k}$.

3. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence the components of $\vec{a}$.

Ans: Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ with $|\vec{a}| = 1$. Then $a_1 = \cos(\pi/3) = 1/2$ and $a_2 = \cos(\pi/4) = 1/\sqrt{2}$. Since $a_1^2 + a_2^2 + a_3^2 = 1$, $(1/2)^2 + (1/\sqrt{2})^2 + \cos^2\theta = 1$, so $1/4 + 1/2 + \cos^2\theta = 1$, giving $\cos^2\theta = 1/4$, so $\cos\theta = 1/2$ (taking the acute angle) and $\theta = \pi/3$. Hence the components of $\vec{a}$ are $(1/2, 1/\sqrt{2}, 1/2)$.

4. Show that $(\vec{a} \times \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

Ans: $(\vec{a} \times \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{b}) \times \vec{b} = \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b} = 0 + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - 0 = 2(\vec{a} \times \vec{b})$.

5. Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

Ans: Expanding the cross product and equating each component to zero gives $6\mu - 27\lambda = 0$, $2\mu - 27 = 0$, and $2\lambda - 6 = 0$. From the last equation $\lambda = 3$, and from the second $\mu = 27/2$. (These values also satisfy the first equation, confirming the solution.)

6. Given that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, what can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

Ans: Since $\vec{a} \cdot \vec{b} = 0$, either $|\vec{a}| = 0$, or $|\vec{b}| = 0$, or $\vec{a}$ and $\vec{b}$ are perpendicular. Since $\vec{a} \times \vec{b} = \vec{0}$, either $|\vec{a}| = 0$, or $|\vec{b}| = 0$, or $\vec{a}$ and $\vec{b}$ are parallel. As the vectors cannot be simultaneously perpendicular and parallel unless one of them is the zero vector, the only possibility is that either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$.

7. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

Ans: Expanding the determinant form of the cross product on both sides component by component shows that $\vec{a} \times (\vec{b} + \vec{c})$ and $\vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ both equal $\hat{i}(a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2) - \hat{j}(a_1b_3 + a_1c_3 - a_3b_1 - a_3c_1) + \hat{k}(a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1)$, confirming the distributive law for the vector product over addition.

8. If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example.

Ans: Take the nonzero vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ (so that $\vec{b} = 2\vec{a}$, making them parallel). Then $\vec{a} \times \vec{b} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = \vec{0}$, even though $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$. So the converse is not true.

9. Find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5).

Ans: $\vec{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{BC} = -\hat{i} + 2\hat{j}$. $\vec{AB} \times \vec{BC} = \hat{i}(0-6) - \hat{j}(0+3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$, with magnitude $\sqrt{(36+9+16)} = \sqrt{61}$. So the area of triangle ABC is $(1/2)\sqrt{61}$ square units.

10. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Ans: $\vec{a} \times \vec{b} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$, with magnitude $\sqrt{(400+25+25)} = \sqrt{450} = 15\sqrt{2}$. So the area of the parallelogram is $15\sqrt{2}$ square units.

11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \sqrt{2}/3$. If $\vec{a} \times \vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is: (A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

Ans: Since $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 1$, $3 \cdot (\sqrt{2}/3) \cdot \sin\theta = 1$, so $\sin\theta = 1/\sqrt{2}$, giving $\theta = \pi/4$. The answer is (B).

12. Area of a rectangle having vertices A, B, C and D with position vectors $\hat{i} + (1/2)\hat{j} + 4\hat{k}$, $\hat{i} + (1/2)\hat{j} + 4\hat{k}$, $\hat{i} - (1/2)\hat{j} + 4\hat{k}$ and $-\hat{i} - (1/2)\hat{j} + 4\hat{k}$ respectively is: (A) $1/2$ (B) 1 (C) 2 (D) 4

Ans: The adjacent sides of rectangle ABCD are $\vec{AB} = 2\hat{i}$ (the i-component increases by 2 from A to B, while j and k stay fixed) and $\vec{BC} = -\hat{j}$ (the j-component decreases by 1 from B to C). So $\vec{AB} \times \vec{BC} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-2-0) = -2\hat{k}$, with magnitude 2. So the area of the rectangle ABCD is 2 square units. The answer is (C).

Miscellaneous Exercise Solutions (All 19 Questions)

1. Write down a unit vector in the XY-plane, making an angle of 30° with the positive direction of the x-axis.

Ans: A unit vector in the XY-plane making an angle θ with the positive x-axis is $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$. For $\theta = 30^\circ$, $\vec{r} = \cos 30^\circ\hat{i} + \sin 30^\circ\hat{j} = (\sqrt{3}/2)\hat{i} + (1/2)\hat{j}$.

2. Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

Ans: $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$. So the scalar components are $(x_2 - x_1)$, $(y_2 - y_1)$, $(z_2 - z_1)$, and $|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

3. A girl walks 4 km towards the west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Ans: Let the starting point be O, with $\vec{OA} = -4\hat{i}$ (4 km west). The second leg AB makes an angle of 30° east of north, i.e. 60° with the positive x-axis, so $\vec{AB} = 3\cos 60^\circ\hat{i} + 3\sin 60^\circ\hat{j} = (3/2)\hat{i} + (3\sqrt{3}/2)\hat{j}$. The total displacement is $\vec{OB} = \vec{OA} + \vec{AB} = (-4 + 3/2)\hat{i} + (3\sqrt{3}/2)\hat{j} = (-5/2)\hat{i} + (3\sqrt{3}/2)\hat{j}$.

4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Ans: In a triangle formed by the vectors, $\vec{a} = \vec{b} + \vec{c}$ by the triangle law of vector addition, but by the triangle inequality for lengths, $|\vec{a}| < |\vec{b}| + |\vec{c}|$ in general (equality holds only when $\vec{b}$ and $\vec{c}$ are in the same direction). So it is not true in general that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.

5. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Ans: $|x(\hat{i} + \hat{j} + \hat{k})| = 1$ gives $\sqrt{x^2 + x^2 + x^2} = 1$, so $\sqrt{3}|x| = 1$, giving $x = \pm 1/\sqrt{3}$.

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Ans: The resultant is $\vec{c} = \vec{a} + \vec{b} = 3\hat{i} + \hat{j}$, with $|\vec{c}| = \sqrt{10}$. So the required vector of magnitude 5, parallel to $\vec{c}$, is $\pm 5(\vec{c}/|\vec{c}|) = \pm(3\sqrt{10}/2)\hat{i} \pm (\sqrt{10}/2)\hat{j}$.

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Ans: $2\vec{a} - \vec{b} + 3\vec{c} = (2-2+3)\hat{i} + (2+1-6)\hat{j} + (2-3+3)\hat{k} = 3\hat{i} - 3\hat{j} + 2\hat{k}$. Its magnitude is $\sqrt{9+9+4} = \sqrt{22}$. So the required unit vector is $(3/\sqrt{22})\hat{i} - (3/\sqrt{22})\hat{j} + (2/\sqrt{22})\hat{k}$.

8. Show that the points A(1,-2,-8), B(5,0,-2) and C(11,3,7) are collinear, and find the ratio in which B divides AC.

Ans: $\vec{AB} = 4\hat{i} + 2\hat{j} + 6\hat{k}$, $\vec{BC} = 6\hat{i} + 3\hat{j} + 9\hat{k}$, $\vec{AC} = 10\hat{i} + 5\hat{j} + 15\hat{k}$. $|\vec{AB}| = 2\sqrt{14}$, $|\vec{BC}| = 3\sqrt{14}$, $|\vec{AC}| = 5\sqrt{14}$, and since $|\vec{AC}| = |\vec{AB}| + |\vec{BC}|$, the points are collinear. If B divides AC in the ratio $\lambda:1$, then $\vec{OB} = (\lambda\vec{OC} + \vec{OA})/(\lambda+1)$. Comparing the x-components, $5(\lambda+1) = 11\lambda+1$, which gives $\lambda = 2/3$. So B divides AC in the ratio 2:3.

9. Find the position vector of a point R which divides the line joining two points P and Q, whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$, externally in the ratio 1:2. Also show that P is the

midpoint of the line segment RQ.

Ans: $\vec{OP} = 2\vec{a} + \vec{b}$, $\vec{OQ} = \vec{a} - 3\vec{b}$. For external division in the ratio 1:2, $\vec{OR} = [2 \cdot \vec{OP} - \vec{OQ}] / (2-1) = 2(2\vec{a} + \vec{b}) - (\vec{a} - 3\vec{b}) = 3\vec{a} + 5\vec{b}$. So the position vector of R is $3\vec{a} + 5\vec{b}$. The midpoint of RQ has position vector $[(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})] / 2 = (4\vec{a} + 2\vec{b}) / 2 = 2\vec{a} + \vec{b} = \vec{OP}$. So P is indeed the midpoint of RQ.

10. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also find its area.

Ans: Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$. The diagonal is $\vec{a} + \vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, with magnitude $\sqrt{9+36+4} = 7$. So the unit vector parallel to the diagonal is $(3/7)\hat{i} - (6/7)\hat{j} + (2/7)\hat{k}$. For the area, $\vec{a} \times \vec{b} = \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4) = 22\hat{i} + 11\hat{j} = 11(2\hat{i} + \hat{j})$, with magnitude $11\sqrt{5}$. So the area of the parallelogram is $11\sqrt{5}$ square units.

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$.

Ans: Let a vector be equally inclined to OX, OY, OZ at angle α . Its direction cosines are $\cos\alpha, \cos\alpha, \cos\alpha$. Since $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$, $3\cos^2\alpha = 1$, giving $\cos\alpha = 1/\sqrt{3}$. So the direction cosines are $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$.

12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

Ans: Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$. Since $\vec{d} \cdot \vec{a} = 0$, $d_1 + 4d_2 + 2d_3 = 0$. Since $\vec{d} \cdot \vec{b} = 0$, $3d_1 - 2d_2 + 7d_3 = 0$. Since $\vec{c} \cdot \vec{d} = 15$, $2d_1 - d_2 + 4d_3 = 15$. Solving these three equations simultaneously gives $d_1 = 160/3$, $d_2 = -5/3$, $d_3 = -70/3$. So $\vec{d} = (1/3)(160\hat{i} - 5\hat{j} - 70\hat{k})$.

13. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

Ans: The sum is $(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$, with magnitude $\sqrt{(\lambda^2 + 4\lambda + 44)}$. The unit vector along the sum is $[(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}] / \sqrt{(\lambda^2 + 4\lambda + 44)}$. Taking the scalar product with $\hat{i} + \hat{j} + \hat{k}$ and setting it equal to 1: $[(2+\lambda) + 6 - 2] / \sqrt{(\lambda^2 + 4\lambda + 44)} = 1$, so $\lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)}$. Squaring, $\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$, so $8\lambda = 8$, giving $\lambda = 1$.

14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

Ans: Since $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$. Let $\theta_1, \theta_2, \theta_3$ be the angles between $\vec{a} + \vec{b} + \vec{c}$ and $\vec{a}, \vec{b}, \vec{c}$ respectively. Then $\cos\theta_1 = (\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a} / (|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|) = |\vec{a}|^2 / (|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|) = |\vec{a}| / |\vec{a} + \vec{b} + \vec{c}|$, and similarly $\cos\theta_2 = |\vec{b}| / |\vec{a} + \vec{b} + \vec{c}|$ and $\cos\theta_3 = |\vec{c}| / |\vec{a} + \vec{b} + \vec{c}|$. Since $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$, so $\theta_1 = \theta_2 = \theta_3$, i.e. $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

15. Prove that $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$.

Ans: $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$. If this equals $|\vec{a}|^2 + |\vec{b}|^2$, then $2\vec{a} \cdot \vec{b} = 0$, so $\vec{a} \cdot \vec{b} = 0$, meaning $\vec{a}$ and $\vec{b}$ are perpendicular. Conversely, if $\vec{a}$ and $\vec{b}$ are perpendicular, $\vec{a} \cdot \vec{b} = 0$, so $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$. This proves the "if and only if" statement.

16. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when: (A) $0 < \theta < \pi/2$ (B) $0 \leq \theta \leq \pi/2$ (C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$

Ans: $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta \geq 0$ requires $\cos\theta \geq 0$ (since $|\vec{a}|, |\vec{b}| \geq 0$), which holds exactly when $0 \leq \theta \leq \pi/2$. The answer is (B).

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if: (A) $\theta = \pi/4$ (B) $\theta = \pi/3$ (C) $\theta = \pi/2$ (D) $\theta = 2\pi/3$

Ans: $|\vec{a} + \vec{b}| = 1$ gives $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$, so $|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 1$. Since $|\vec{a}| = |\vec{b}| = 1$, $1 + 2\cos\theta + 1 = 1$, giving $\cos\theta = -1/2$, so $\theta = 2\pi/3$. The answer is (D).

18. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is: (A) 0 (B) -1 (C) 1 (D) 3

Ans: Since $\hat{j} \times \hat{k} = \hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$, and $\hat{i} \times \hat{j} = \hat{k}$, the expression becomes $\hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k} = 1 - 1 + 1 = 1$. The answer is (C).

19. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to: (A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Ans: $|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos\theta$ and $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta$. Setting these equal gives $\cos\theta = \sin\theta$, so $\tan\theta = 1$, giving $\theta = \pi/4$. The answer is (B).

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