

CLASS 12

MATHEMATICS

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NCERT Solutions for Class 12 Mathematics Chapter 11: Three-Dimensional Geometry – Free PDF Download

Chapter 11 of Class 12 Maths is Three-Dimensional Geometry. It extends vector concepts to lines and planes in 3D space, covering direction cosines/ratios, equations of lines and planes, and distances/angles between them.

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Chapter 11 of Class 12 Maths is Three-Dimensional Geometry. It extends vector concepts to lines and planes in 3D space, covering direction cosines/ratios, equations of lines and planes, and distances/angles between them.

Direction Cosines and Direction Ratios

If a line makes angles α, β, γ with the x, y, z axes, $\cos\alpha, \cos\beta, \cos\gamma$ are its **direction cosines** (l, m, n), satisfying $l^2 + m^2 + n^2 = 1$. Any numbers proportional to l, m, n are **direction ratios**.

Equation of a Line in Space

Vector form: $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ (through point with position vector $\mathbf{a}$, along direction $\mathbf{b}$). **Cartesian form:** $(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$.

Angle Between Two Lines

Using direction ratios: $\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / [\sqrt{(a_1^2 + b_1^2 + c_1^2)}\sqrt{(a_2^2 + b_2^2 + c_2^2)}]$. Lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Equation of a Plane

Vector form (normal form): $\mathbf{r} \cdot \mathbf{n} = d$. **Cartesian form:** $ax + by + cz = d$, where a, b, c are direction ratios of the normal.

Distances

Distance of a point from a plane: $|ax_1 + by_1 + cz_1 - d| / \sqrt{a^2 + b^2 + c^2}$. **Shortest distance between skew lines** is computed via the scalar triple product formula.

Exercise 11.1 Solutions (All 5 Questions)

1. If a line makes angles $90^\circ, 135^\circ, 45^\circ$ with the x, y and z axes respectively, find its direction cosines.

Ans: Let the direction cosines of the line be l, m, n . Then $l = \cos 90^\circ = 0$, $m = \cos 135^\circ = -1/\sqrt{2}$, $n = \cos 45^\circ = 1/\sqrt{2}$. So the direction cosines are $0, -1/\sqrt{2}, 1/\sqrt{2}$.

2. Find the direction cosines of a line which makes equal angles with the coordinate axes.

Ans: Let the line make an angle α with each of the coordinate axes, so $l = m = n = \cos\alpha$. Since $l^2 + m^2 + n^2 = 1$, $3\cos^2\alpha = 1$, giving $\cos\alpha = \pm 1/\sqrt{3}$. So the direction cosines are $\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3}$.

3. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

Ans: Here $a = -18, b = 12, c = -4$. $\sqrt{(a^2 + b^2 + c^2)} = \sqrt{(324 + 144 + 16)} = \sqrt{484} = 22$. So the direction cosines are $l = a/22 = -18/22 = -9/11$, $m = b/22 = 12/22 = 6/11$, $n = c/22 = -4/22 = -2/11$.

4. Show that the points $(2,3,4), (-1,-2,1), (5,8,7)$ are collinear.

Ans: Let $A(2,3,4), B(-1,-2,1), C(5,8,7)$. The direction ratios of AB are $(-1-2, -2-3, 1-4) = (-3, -5, -3)$. The direction ratios of BC are $(5-(-1), 8-(-2), 7-1) = (6, 10, 6)$. Since $BC = -2 \cdot AB$, the direction ratios of AB and BC are proportional, so AB is parallel to BC . As point B is common to both, the points A, B, C are collinear.

5. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2).

Ans: Let A(3,5,-4), B(-1,1,2), C(-5,-5,-2). The direction ratios of AB are $(-1-3, 1-5, 2-(-4)) = (-4, -4, 6)$, with magnitude $\sqrt{16+16+36} = 2\sqrt{17}$, giving direction cosines $(-2/\sqrt{17}, -2/\sqrt{17}, 3/\sqrt{17})$. The direction ratios of BC are $(-5-(-1), -5-1, -2-2) = (-4, -6, -4)$, with magnitude $\sqrt{16+36+16} = 2\sqrt{17}$, giving direction cosines $(-2/\sqrt{17}, -3/\sqrt{17}, -2/\sqrt{17})$. The direction ratios of AC are $(-5-3, -5-5, -2-(-4)) = (-8, -10, 2)$, with magnitude $\sqrt{64+100+4} = 2\sqrt{42}$, giving direction cosines $(-4/\sqrt{42}, -5/\sqrt{42}, 1/\sqrt{42})$.

Exercise 11.2 Solutions (All 15 Questions)

1. Show that the three lines with direction cosines $12/13, -3/13, -4/13$; $4/13, 12/13, 3/13$; $3/13, -4/13, 12/13$ are mutually perpendicular.

Ans: Two lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are perpendicular to each other if $l_1l_2 + m_1m_2 + n_1n_2 = 0$.

(i) For $12/13, -3/13, -4/13$ and $4/13, 12/13, 3/13$: $(12/13)(4/13) + (-3/13)(12/13) + (-4/13)(3/13) = 48/169 - 36/169 - 12/169 = 0$. So these lines are perpendicular.

(ii) For $4/13, 12/13, 3/13$ and $3/13, -4/13, 12/13$: $(4/13)(3/13) + (12/13)(-4/13) + (3/13)(12/13) = 12/169 - 48/169 + 36/169 = 0$. So these lines are perpendicular.

(iii) For $3/13, -4/13, 12/13$ and $12/13, -3/13, -4/13$: $(3/13)(12/13) + (-4/13)(-3/13) + (12/13)(-4/13) = 36/169 + 12/169 - 48/169 = 0$. So these lines are perpendicular.

Hence, all three lines are mutually perpendicular.

2. Show that the line through the points (1,-1,2) and (3,4,-2) is perpendicular to the line through the points (0,3,2) and (3,5,6).

Ans: Let AB be the line joining (1,-1,2) and (3,4,-2), and CD be the line joining (0,3,2) and (3,5,6). The direction ratios of AB are $(3-1, 4-(-1), -2-2) = (2, 5, -4)$. The direction ratios of CD are $(3-0, 5-3, 6-2) = (3, 2, 4)$. AB and CD are perpendicular if the sum of the products of corresponding direction ratios is zero: $(2 \times 3) + (5 \times 2) + (-4 \times 4) = 6 + 10 - 16 = 0$. Therefore, AB and CD are perpendicular to each other.

3. Show that the line through the points (4,7,8), (2,3,4) is parallel to the line through the points (-1,-2,1), (1,2,5).

Ans: Let AB be the line through (4,7,8) and (2,3,4), and CD be the line through (-1,-2,1) and (1,2,5). The direction ratios of AB are $(2-4, 3-7, 4-8) = (-2, -4, -4)$. The direction ratios of CD are $(1-(-1), 2-(-2), 5-1) = (2, 4, 4)$. Since AB's direction ratios are -1 times CD's direction ratios, they are proportional: $-2/2 = -4/4 = -4/4 = -1$. Since the direction ratios of AB and CD are proportional, AB is parallel to CD.

4. Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

Ans: The line passes through the point A(1,2,3), so the position vector of A is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. Also, $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$. The line through point A and parallel to $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$. So $\vec{r} = (1+2\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3-2\lambda)\hat{k}$, which is the required equation of the line.

5. Find the equation of the line in vector and Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

Ans: The line passes through the point with position vector $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$, and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. The line through this point and parallel to $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, so $\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which gives $\vec{r} = (2+\lambda)\hat{i} + (-1+2\lambda)\hat{j} + (4-\lambda)\hat{k}$. This is the required equation in vector form. Eliminating λ , we obtain the Cartesian form: $(x-2)/1 = (y+1)/2 = (z-4)/-1$.

6. Find the Cartesian equation of the line which passes through the point (-2,4,-5) and is parallel to the line given by $(x+3)/3 = (y-4)/5 = (z+8)/6$.

Ans: The required line passes through (-2,4,-5) and is parallel to $(x+3)/3 = (y-4)/5 = (z+8)/6$, whose

direction ratios are 3,5,6. Since the required line is parallel to this line, its direction ratios are $3k, 5k, 6k$ for some $k \neq 0$. The equation of a line through (x_1, y_1, z_1) with direction ratios (a, b, c) is $(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$. So the equation of the required line is $(x+2)/3k = (y-4)/5k = (z+5)/6k$, which simplifies to $(x+2)/3 = (y-4)/5 = (z+5)/6$.

7. The Cartesian equation of a line is $(x-5)/3 = (y+4)/7 = (z-6)/2$. Write its vector form.

Ans: The given line passes through the point $(5, -4, 6)$, whose position vector is $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$. The direction ratios of the line are 3, 7, 2, so the line is in the direction of $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$. The line through point A and parallel to $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, so $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$, which is the required equation of the line in vector form.

8. Find the angle between the following pairs of lines: (i) $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$ and $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$; (ii) $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$.

Ans: The angle Q between two lines parallel to vectors $\vec{b}_1$ and $\vec{b}_2$ is given by $\cos Q = |\vec{b}_1 \cdot \vec{b}_2| / (|\vec{b}_1| |\vec{b}_2|)$.

(i) Here $\vec{b}_1 = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$. $|\vec{b}_1| = \sqrt{9+4+36} = 7$ and $|\vec{b}_2| = \sqrt{1+4+4} = 3$. $\vec{b}_1 \cdot \vec{b}_2 = 3 \times 1 + (-2) \times 2 + 6 \times 2 = 3 - 4 + 12 = 11$. So $\cos Q = 11/21$, giving $Q = \cos^{-1}(11/21)$.

(ii) Here $\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$. $|\vec{b}_1| = \sqrt{1+1+4} = \sqrt{6}$ and $|\vec{b}_2| = \sqrt{9+25+16} = \sqrt{50} = 5\sqrt{2}$. $\vec{b}_1 \cdot \vec{b}_2 = 1 \times 3 + (-1) \times (-5) + (-2) \times (-4) = 3 + 5 + 8 = 16$. So $\cos Q = 16/(\sqrt{6} \times 5\sqrt{2}) = 16/(5\sqrt{12}) = 8/(5\sqrt{3})$, giving $Q = \cos^{-1}(8/(5\sqrt{3}))$.

9. Find the angle between the following pairs of lines: (i) $(x-2)/2 = (y-1)/5 = (z+3)/-3$ and $(x+2)/-1 = (y-4)/8 = (z-5)/4$; (ii) $x/2 = y/2 = z/1$ and $(x-5)/-4 = (y-2)/1 = (z-3)/8$.

Ans: The angle Q between two lines with direction vectors $\vec{b}_1$ and $\vec{b}_2$ is given by $\cos Q = |\vec{b}_1 \cdot \vec{b}_2| / (|\vec{b}_1| |\vec{b}_2|)$.

(i) Here $\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}$ and $\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$. $|\vec{b}_1| = \sqrt{4+25+9} = \sqrt{38}$ and $|\vec{b}_2| = \sqrt{1+64+16} = 9$. $\vec{b}_1 \cdot \vec{b}_2 = 2 \times (-1) + 5 \times 8 + (-3) \times 4 = -2 + 40 - 12 = 26$. So $\cos Q = 26/(9\sqrt{38})$, giving $Q = \cos^{-1}(26/(9\sqrt{38}))$.

(ii) Here $\vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b}_2 = -4\hat{i} + \hat{j} + 8\hat{k}$. $|\vec{b}_1| = \sqrt{4+4+1} = 3$ and $|\vec{b}_2| = \sqrt{16+1+64} = 9$. $\vec{b}_1 \cdot \vec{b}_2 = 2 \times (-4) + 2 \times 1 + 1 \times 8 = -8 + 2 + 8 = 2$. So $\cos Q = 2/(3 \times 9) = 2/27$, giving $Q = \cos^{-1}(2/27)$.

10. Find the value of p so that the lines $(1-x)/3 = (7y-14)/2p = (z-3)/2$ and $(7-7x)/3p = (y-5)/1 = (6-z)/5$ are at right angles.

Ans: Writing the given equations in standard form: $(x-1)/-3 = (y-2)/(2p/7) = (z-3)/2$ and $(x-1)/(-3p/7) = (y-5)/1 = (z-6)/-5$. So the direction ratios of the two lines are -3, $2p/7$, 2 and $-3p/7$, 1, -5. Two lines are perpendicular if the sum of the products of corresponding direction ratios is zero:

$(-3)(-3p/7) + (2p/7)(1) + (2)(-5) = 0$, which gives $9p/7 + 2p/7 - 10 = 0$, so $11p/7 = 10$, giving $11p = 70$, so $p = 70/11$.

11. Show that the lines $(x-5)/7 = (y+2)/-5 = z/1$ and $x/1 = y/2 = z/3$ are perpendicular to each other.

Ans: The direction ratios of the given lines are 7, -5, 1 and 1, 2, 3. Two lines are perpendicular if the sum of the products of corresponding direction ratios is zero: $7 \times 1 + (-5) \times 2 + 1 \times 3 = 7 - 10 + 3 = 0$. Therefore, the given lines are perpendicular to each other.

12. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

Ans: Comparing with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, we get $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$. So $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$. Also, $\vec{b}_1 \times \vec{b}_2 = \hat{i}(-1 \times 2 - 1 \times 1) - \hat{j}(1 \times 2 - 1 \times 2) + \hat{k}(1 \times 1 - (-1) \times 2) = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$, with magnitude $\sqrt{9+9} = 3\sqrt{2}$. The shortest distance is given by $d = |(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)| / |\vec{b}_1 \times \vec{b}_2| = |(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})| / 3\sqrt{2} = |(-3)(1) + 3(-2)| / 3\sqrt{2} = |-9| / 3\sqrt{2} = 3/\sqrt{2}$. Therefore, the shortest distance between the given lines is $3/\sqrt{2}$ units.

13. Find the shortest distance between the lines $(x+1)/7 = (y+1)/-6 = (z+1)/1$ and $(x-3)/1 = (y-5)/-2 =$

(z-7)/1.

Ans: The shortest distance between the lines $(x-x_1)/a_1 = (y-y_1)/b_1 = (z-z_1)/c_1$ and $(x-x_2)/a_2 = (y-y_2)/b_2 = (z-z_2)/c_2$ is given by the formula $d = |\text{determinant}| / \sqrt{((b_1c_2-b_2c_1)^2 + (c_1a_2-c_2a_1)^2 + (a_1b_2-a_2b_1)^2)}$, where the determinant has rows $(x_2-x_1, y_2-y_1, z_2-z_1), (a_1, b_1, c_1), (a_2, b_2, c_2)$. Here $x_1=-1, y_1=-1, z_1=-1, a_1=7, b_1=-6, c_1=1, x_2=3, y_2=5, z_2=7, a_2=1, b_2=-2, c_2=1$. So the determinant with rows $(4, 6, 8), (7, -6, 1), (1, -2, 1)$ equals $4(-6+2)-6(7-1)+8(-14+6) = -16-36-64 = -116$. Also, $\sqrt{((-6+2)^2 + (1-7)^2 + (-14+6)^2)} = \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$. So $d = |-116|/2\sqrt{29} = 58/\sqrt{29} = 2\sqrt{29}$. Since distance is always non-negative, the shortest distance between the given lines is $2\sqrt{29}$ units.

14. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1+2\hat{j}+3\hat{k})+\lambda(1-3\hat{j}+2\hat{k})$ and $\vec{r} = (4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

Ans: Comparing with the standard form, $\vec{a} \rightarrow_1 = \hat{i}+2\hat{j}+3\hat{k}, \vec{b} \rightarrow_1 = \hat{i}-3\hat{j}+2\hat{k}, \vec{a} \rightarrow_2 = 4\hat{i}+5\hat{j}+6\hat{k}, \vec{b} \rightarrow_2 = 2\hat{i}+3\hat{j}+\hat{k}$. So $\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1 = 3\hat{i}+3\hat{j}+3\hat{k}$. Also, $\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2 = \hat{i}(-3 \times 1 - 2 \times 3) - \hat{j}(1 \times 1 - 2 \times 2) + \hat{k}(1 \times 3 - (-3) \times 2) = \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6) = -9\hat{i}+3\hat{j}+9\hat{k}$, with magnitude $\sqrt{(81+9+81)} = \sqrt{171} = 3\sqrt{19}$. The shortest distance is $d = |(\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2) \cdot (\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1)| / |\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2| = |(-9)(3) + (3)(3) + (9)(3)| / 3\sqrt{19} = |9| / 3\sqrt{19} = 3/\sqrt{19}$. Therefore, the shortest distance between the given lines is $3/\sqrt{19}$ units.

15. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$.

Ans: Rewriting the first equation: $\vec{r} = \hat{i}-t\hat{i}+t\hat{j}-2\hat{j}+3\hat{k}-2t\hat{k} = (\hat{i}-2\hat{j}+3\hat{k})+t(-\hat{i}+\hat{j}-2\hat{k})$. Rewriting the second equation: $\vec{r} = s\hat{i}+\hat{i}+2s\hat{j}-\hat{j}-2s\hat{k}-\hat{k} = (\hat{i}-\hat{j}-\hat{k})+s(\hat{i}+2\hat{j}-2\hat{k})$. So $\vec{a} \rightarrow_1 = \hat{i}-2\hat{j}+3\hat{k}, \vec{b} \rightarrow_1 = -\hat{i}+\hat{j}-2\hat{k}, \vec{a} \rightarrow_2 = \hat{i}-\hat{j}-\hat{k}, \vec{b} \rightarrow_2 = \hat{i}+2\hat{j}-2\hat{k}$. So $\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1 = \hat{j}-4\hat{k}$. Also, $\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2 = \hat{i}(1 \times (-2) - (-2) \times 2) - \hat{j}((-1) \times (-2) - (-2) \times 1) + \hat{k}((-1) \times 2 - 1 \times 1) = \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1) = 2\hat{i}-4\hat{j}-3\hat{k}$, with magnitude $\sqrt{(4+16+9)} = \sqrt{29}$. The shortest distance is $d = |(\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2) \cdot (\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1)| / |\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2| = |(2)(0) + (-4)(1) + (-3)(-4)| / \sqrt{29} = |8| / \sqrt{29} = 8/\sqrt{29}$. Therefore, the shortest distance between the given lines is $8/\sqrt{29}$ units.

Miscellaneous Exercise Solutions (All 5 Questions)

1. Find the angle between the lines whose direction ratios are a, b, c and b-c, c-a, a-b.

Ans: Let θ be the angle between the lines with direction ratios a, b, c and b-c, c-a, a-b. The cosine of the angle between two lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2) is given by $\cos\theta = |a_1a_2+b_1b_2+c_1c_2| / (\sqrt{a_1^2+b_1^2+c_1^2} \cdot \sqrt{a_2^2+b_2^2+c_2^2})$. Here the numerator is $a(b-c)+b(c-a)+c(a-b) = ab-ac+bc-ab+ac-bc = 0$. So $\cos\theta = 0$, which gives $\theta = \cos^{-1}0 = 90^\circ$. Therefore, the angle between the two lines is 90° .

2. Find the equation of a line parallel to the x-axis and passing through the origin.

Ans: A line parallel to the x-axis and passing through the origin is the x-axis itself. Let A be a point on the x-axis, so the coordinates of A are (a, 0, 0). The direction ratios of OA are $(a-0, 0-0, 0-0) = (a, 0, 0)$, i.e., proportional to 1, 0, 0. So the equation of OA is $(x-0)/1 = (y-0)/0 = (z-0)/0$. Therefore, the equation of the line passing through the origin and parallel to the x-axis is $x/1 = y/0 = z/0$.

3. If the lines $(x-1)/-3 = (y-2)/2k = (z-3)/2$ and $(x-1)/3k = (y-1)/1 = (z-6)/-5$ are perpendicular, find the value of k.

Ans: From the given equations, the direction ratios of the two lines are $a_1=-3, b_1=2k, c_1=2$ and $a_2=3k, b_2=1, c_2=-5$. The two lines are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$. So $-3(3k)+2k(1)+2(-5) = 0$, which gives $-9k+2k-10 = 0$, so $-7k = 10$, giving $k = -10/7$.

4. Find the shortest distance between the lines $\vec{r} = 6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r} = -4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

Ans: Comparing with $\vec{r} = \vec{a} \rightarrow_1 + \lambda \vec{b} \rightarrow_1$ and $\vec{r} = \vec{a} \rightarrow_2 + \mu \vec{b} \rightarrow_2$, we get $\vec{a} \rightarrow_1 = 6\hat{i}+2\hat{j}+2\hat{k}, \vec{b} \rightarrow_1 = \hat{i}-2\hat{j}+2\hat{k}, \vec{a} \rightarrow_2 = -4\hat{i}-\hat{k}, \vec{b} \rightarrow_2 = 3\hat{i}-2\hat{j}-2\hat{k}$. So $\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1 = 10\hat{i}-2\hat{j}-3\hat{k}$. Also, $\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2 = \hat{i}((-2)(-2)-(2)(-2)) - \hat{j}((1)(-2)-(2)(3)) + \hat{k}((1)(-2)-(-2)(3)) = \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6) = 8\hat{i}+8\hat{j}+4\hat{k}$, with magnitude $\sqrt{(64+64+16)} = \sqrt{144} = 12$. The shortest distance is given by $d = |(\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2) \cdot (\vec{a} \rightarrow_2 - \vec{a} \rightarrow_1)| / |\vec{b} \rightarrow_1 \times \vec{b} \rightarrow_2| =$

$| (8)(10) + (8)(2) + (4)(3) | / 12 = | 80 + 16 + 12 | / 12 = 108 / 12 = 9$. Therefore, the shortest distance between the given lines is 9 units.

5. Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines $(x-8)/3 = (y+19)/-16 = (z-10)/7$ and $(x-15)/3 = (y-29)/8 = (z-5)/-5$.

Ans: Let the required line be parallel to $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, and let $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$ be the position vector of the given point. The equation of the required line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \dots (1)$. The direction ratios of the two given lines are 3, -16, 7 and 3, 8, -5. Since the required line is perpendicular to both of these lines: $3b_1 - 16b_2 + 7b_3 = 0 \dots (2)$, and $3b_1 + 8b_2 - 5b_3 = 0 \dots (3)$. Solving (2) and (3) using cross-multiplication: $b_1 / ((-16)(-5) - (8)(7)) = b_2 / ((7)(3) - (3)(-5)) = b_3 / ((3)(8) - (3)(-16))$, which gives $b_1/24 = b_2/36 = b_3/72$, i.e., $b_1/2 = b_2/3 = b_3/6$. So the direction ratios of $\vec{b}$ are 2, 3, 6, giving $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Substituting this into (1), the required vector equation of the line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$.

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