

CLASS 12

MATHEMATICS

NCERT SOLUTIONS

NCERT Solutions for Class 12 Mathematics Chapter 13: Probability – Free PDF Download

Chapter 13 of Class 12 Maths is Probability, the final chapter of the syllabus. It covers conditional probability, multiplication theorem, independent events, Bayes theorem, and probability distributions.

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Chapter 13 of Class 12 Maths is Probability, the final chapter of the syllabus. It covers conditional probability, multiplication theorem, independent events, Bayes theorem, and probability distributions.

Conditional Probability

The **conditional probability** of event A given event B has occurred is $P(A|B) = P(A \cap B)/P(B)$, provided $P(B) > 0$.

Multiplication Theorem

$P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$. Extended to three events: $P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$.

Independent Events

Events A and B are **independent** if $P(A \cap B) = P(A) \cdot P(B)$, equivalently $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Total Probability Theorem and Bayes Theorem

If $E_1, E_2, \dots, E_n$ are mutually exclusive and exhaustive events (a partition of the sample space), the **law of total probability** gives $P(A) = \sum P(E_i)P(A|E_i)$. **Bayes theorem** reverses conditioning:

$$P(E_i|A) = [P(E_i)P(A|E_i)] / [\sum P(E_j)P(A|E_j)].$$

Random Variables and Probability Distributions

A **random variable** X assigns a numerical value to each outcome. Its **probability distribution** lists all values with their probabilities (summing to 1). **Mean** $E(X) = \sum x_i p_i$; **Variance** $\text{Var}(X) = E(X^2) - [E(X)]^2$.

Bernoulli Trials and Binomial Distribution

Bernoulli trials are repeated independent trials with only two outcomes (success/failure). The **binomial distribution** gives $P(X=r) = {}^n C_r p^r q^{n-r}$, where p=probability of success, $q=1-p$.

Exercise 13.1 Solutions (All 17 Questions)

1. Given that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$.

Ans: $P(E|F) = P(E \cap F)/P(F) = 0.2/0.3 = 2/3$. $P(F|E) = P(E \cap F)/P(E) = 0.2/0.6 = 1/3$.

2. Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

Ans: $P(A|B) = P(A \cap B)/P(B) = 0.32/0.5 = 0.64$.

3. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find (i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$.

Ans: (i) $P(A \cap B) = P(B|A) \times P(A) = 0.4 \times 0.8 = 0.32$. (ii) $P(A|B) = P(A \cap B)/P(B) = 0.32/0.5 = 0.64$. (iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.5 - 0.32 = 0.98$.

4. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = 5/13$ and $P(A|B) = 2/5$.

Ans: From $2P(A) = 5/13$, $P(A) = 5/26$, and $P(B) = 5/13$. Also $P(A \cap B) = P(A|B) \times P(B) = (2/5) \times (5/13) = 2/13$. So $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 5/26 + 5/13 - 2/13 = 5/26 + 10/26 - 4/26 = 11/26$.

5. If $P(A) = 6/11$, $P(B) = 5/11$ and $P(A \cup B) = 7/11$, find (i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(B|A)$.

Ans: (i) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 6/11 + 5/11 - 7/11 = 4/11$. (ii) $P(A|B) = P(A \cap B)/P(B) = (4/11)/(5/11) = 4/5$. (iii) $P(B|A) = P(A \cap B)/P(A) = (4/11)/(6/11) = 2/3$.

6. A coin is tossed three times. Find $P(E|F)$ in each case: (i) E = head on third toss, F = heads on first two tosses (ii) E = at least two heads, F = at most two heads (iii) E = at most two tails, F = at least one tail.

Ans: The sample space has 8 equally likely outcomes. (i) $F = \{HHH, HHT\}$, and among these E (head on 3rd toss) occurs only for HHH . So $P(E|F) = 1/2$. (ii) $F = \{\text{all outcomes except } HHH\}$ has 7 outcomes; among these, E (at least 2 heads) occurs for HHT, HTH, THH , i.e. 3 outcomes. So $P(E|F) = 3/7$. (iii) $F = \{\text{all outcomes except } HHH\}$ has 7 outcomes; among these, E (at most 2 tails) occurs for all except TTT , i.e. 6 outcomes. So $P(E|F) = 6/7$.

7. Two coins are tossed once. Find $P(E|F)$ if E : tail appears on one coin, F : one coin shows head.

Ans: The sample space is $\{HH, HT, TH, TT\}$. F (one coin shows head) = $\{HT, TH\}$. E (tail on one coin) = $\{HT, TH\}$. So $E \cap F = \{HT, TH\}$, and $P(E|F) = P(E \cap F)/P(F) = (2/4)/(2/4) = 1$.

8. A die is thrown three times. Find $P(E|F)$ where E : 4 appears on the third toss, F : 6 and 5 appear respectively on first two tosses.

Ans: F occurs only when the first throw is 6 and the second is 5, so there are 6 outcomes in F (one for each possible third throw). Among these, E (4 on third toss) occurs for exactly 1 outcome. So $P(E|F) = 1/6$.

9. Mother, father and son line up at random for a family photo. Find $P(E|F)$ if E : son on one end, F : father in middle.

Ans: There are $3! = 6$ equally likely arrangements. F (father in middle) occurs for 2 of them: (Mother, Father, Son) and (Son, Father, Mother). In both of these, the son is at one end, so $E \cap F$ has 2 outcomes too. Therefore $P(E|F) = 2/2 = 1$.

10. A black and a red die are rolled together. (a) Find the conditional probability of getting a sum greater than 9, given that the black die resulted in a 5. (b) Find the conditional probability of getting the sum 8, given that the red die resulted in a number less than 4.

Ans: (a) Given the black die shows 5, the red die can be 1-6 (6 equally likely outcomes); the sum exceeds 9 (i.e. sum = 10 or 11) for red die = 5 or 6, i.e. 2 outcomes. So the probability is $2/6 = 1/3$. (b) Given the red die shows less than 4 (i.e. 1, 2 or 3), there are $3 \times 6 = 18$ equally likely outcomes for the pair; the sum equals 8 for (black, red) = (7, 1) [invalid], (6, 2), (5, 3), i.e. 2 valid outcomes. So the probability is $2/18 = 1/9$.

11. A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$. Find (i) $P(E|F)$ and $P(F|E)$ (ii) $P(E|G)$ and $P(G|E)$ (iii) $P((E \cup F)|G)$ and $P((E \cap F)|G)$.

Ans: All events have equally likely outcomes out of 6. (i) $E \cap F = \{3\}$, so $P(E|F) = (1/6)/(2/6) = 1/2$, and $P(F|E) = (1/6)/(3/6) = 1/3$. (ii) $E \cap G = \{3, 5\}$, so $P(E|G) = (2/6)/(4/6) = 1/2$, and $P(G|E) = (2/6)/(3/6) = 2/3$. (iii) $E \cup F = \{1, 2, 3, 5\}$, so $(E \cup F) \cap G = \{2, 3, 5\}$, giving $P((E \cup F)|G) = (3/6)/(4/6) = 3/4$. $E \cap F = \{3\}$, so $(E \cap F) \cap G = \{3\}$, giving $P((E \cap F)|G) = (1/6)/(4/6) = 1/4$.

12. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Ans: The total number of questions is $300 + 200 + 500 + 400 = 1400$. Let E denote "the question is easy" and F denote "the question is a multiple choice question". The number of multiple choice questions is $500 + 400 = 900$, so $P(F) = 900/1400$. The number of easy multiple choice questions is 500, so $P(E \cap F) = 500/1400$. Therefore $P(E|F) = P(E \cap F)/P(F) = 500/900 = 5/9$.

13. A family has two children. What is the probability that both children are girls, given that (i) the youngest is a girl (ii) at least one is a girl?

Ans: The sample space (elder, younger) is {BB, BG, GB, GG}, each equally likely. (i) $F = \text{"youngest is a girl"} = \{BG, GG\}$, and $E \cap F = \{GG\}$. So $P(E|F) = (1/4)/(2/4) = 1/2$. (ii) $F = \text{"at least one is a girl"} = \{BG, GB, GG\}$, and $E \cap F = \{GG\}$. So $P(E|F) = (1/4)/(3/4) = 1/3$.

14. Two dice are thrown together, and the events E and F are defined as follows: E: sum of numbers on the two dice is 4, F: the two numbers appearing on the dice are different. Find $P(E|F)$.

Ans: Out of 36 equally likely outcomes, F (dice show different numbers) excludes the 6 outcomes where both dice match, leaving 30 outcomes. Among these, E (sum equals 4) occurs for (1,3) and (3,1) only, i.e. 2 outcomes (since (2,2) is excluded as the dice match). So $P(E|F) = 2/30 = 1/15$.

15. A die is thrown. If it shows a multiple of 3, it is thrown again; otherwise a coin is tossed once. Find the conditional probability of the event "the coin shows a tail", given that "at least one die throw shows a 3".

Ans: Let $E = \text{"the coin shows a tail"}$ and $F = \text{"at least one throw of the die results in a 3"}$. The event F can only occur along the branch where the die was actually thrown (which happens only when the first throw is a multiple of 3, i.e. 3 or 6, triggering a second die throw) — a coin is never tossed in that branch. So whenever F occurs, no coin was ever tossed, which means E (the coin shows a tail) cannot occur. Hence $E \cap F = \emptyset$, so $P(E \cap F) = 0$, and therefore $P(E|F) = P(E \cap F)/P(F) = 0$.

16. If $P(A) = 1/2$, $P(B) = 0$, find $P(A|B)$.

Ans: By definition, $P(A|B) = P(A \cap B)/P(B)$. Since $P(B) = 0$, this expression is not defined (division by zero). Hence $P(A|B)$ is not defined.

17. If A and B are events such that $P(A|B) = P(B|A)$, then which of the following is correct?

Ans: $P(A|B) = P(A \cap B)/P(B)$ and $P(B|A) = P(A \cap B)/P(A)$. If these are equal, then $P(A \cap B)/P(B) = P(A \cap B)/P(A)$. Provided $P(A \cap B) \neq 0$, this simplifies to $P(A) = P(B)$.

Exercise 13.2 Solutions (All 18 Questions)

1. If $P(A) = 3/5$ and $P(B) = 1/5$, find $P(A \cap B)$ if A and B are independent events.

Ans: Since A and B are independent, $P(A \cap B) = P(A) \times P(B) = (3/5) \times (1/5) = 3/25$.

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Ans: There are 26 black cards. The probability the first card is black is $26/52$, and given the first is black, the probability the second is also black is $25/51$. So $P(\text{both black}) = (26/52) \times (25/51) = 25/102$.

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all three are good, the box is approved for sale, otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Ans: The probability that all three drawn oranges are good is $C(12,3)/C(15,3) = 220/455 = 44/91$.

4. A fair coin and an unbiased die are tossed. Let A be the event "head appears on the coin" and B be the event "3 appears on the die". Check whether A and B are independent events or not.

Ans: $P(A) = 1/2$ and $P(B) = 1/6$. Since the coin toss and die throw are physically independent experiments, $P(A \cap B) = P(A) \times P(B) = (1/2) \times (1/6) = 1/12$. Since this equals the product of the individual probabilities, A and B are independent events.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event “the number is even” and B be the event “the number is red”. Are A and B independent?

Ans: $A = \{2, 4, 6\}$, so $P(A) = 3/6 = 1/2$. $B = \{1, 2, 3\}$, so $P(B) = 3/6 = 1/2$. $A \cap B = \{2\}$, so $P(A \cap B) = 1/6$. Since $P(A) \times P(B) = 1/4$, which is not equal to $P(A \cap B) = 1/6$, A and B are not independent.

6. Let E and F be events with $P(E) = 3/5$, $P(F) = 3/10$ and $P(E \cap F) = 1/5$. Are E and F independent?

Ans: $P(E) \times P(F) = (3/5) \times (3/10) = 9/50 = 0.18$. But $P(E \cap F) = 1/5 = 0.2$. Since these are not equal, E and F are not independent.

7. Given that the events A and B are such that $P(A) = 1/2$, $P(A \cup B) = 3/5$ and $P(B) = p$. Find p if they are (i) mutually exclusive (ii) independent.

Ans: (i) If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$, so $3/5 = 1/2 + p$, giving $p = 3/5 - 1/2 = 1/10$.

(ii) If A and B are independent, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $3/5 = 1/2 + p - p/2 = 1/2 + p/2$, giving $p/2 = 1/10$, so $p = 1/5$.

8. Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$. Find (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A|B)$ (iv) $P(B|A)$.

Ans: (i) $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$. (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.4 - 0.12 = 0.58$. (iii) $P(A|B) = P(A \cap B)/P(B) = 0.12/0.4 = 0.3$. (iv) $P(B|A) = P(A \cap B)/P(A) = 0.12/0.3 = 0.4$.

9. If A and B are two events such that $P(A) = 1/4$, $P(B) = 1/2$ and $P(A \cap B) = 1/8$, find $P(\text{not A and not B})$.

Ans: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/4 + 1/2 - 1/8 = 2/8 + 4/8 - 1/8 = 5/8$. By De Morgan's law, $P(A' \cap B') = 1 - P(A \cup B) = 1 - 5/8 = 3/8$.

10. Events A and B are such that $P(A) = 1/2$, $P(B) = 7/12$ and $P(\text{not A or not B}) = 1/4$. State whether A and B are independent.

Ans: Since $P(A' \cup B') = 1/4$, by De Morgan's law $P(A \cap B) = 1 - P(A' \cup B') = 1 - 1/4 = 3/4$. But $P(A) \times P(B) = (1/2) \times (7/12) = 7/24$. Since $P(A \cap B) = 3/4 \neq 7/24 = P(A) \times P(B)$, A and B are not independent.

11. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$. Find (i) $P(A \text{ and } B)$ (ii) $P(A \text{ and not } B)$ (iii) $P(A \text{ or } B)$ (iv) $P(\text{neither A nor B})$.

Ans: (i) $P(A \cap B) = 0.3 \times 0.6 = 0.18$. (ii) $P(A \cap B') = P(A) \times P(B') = 0.3 \times (1 - 0.6) = 0.3 \times 0.4 = 0.12$. (iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = 0.72$. (iv) $P(A' \cap B') = (1 - 0.3) \times (1 - 0.6) = 0.7 \times 0.4 = 0.28$.

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

Ans: The probability of not getting an odd number (i.e. getting an even number) on a single throw is $1/2$. Since the throws are independent, $P(\text{no odd number in 3 throws}) = (1/2)^3 = 1/8$. Therefore $P(\text{at least one odd number}) = 1 - 1/8 = 7/8$.

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both balls are red (ii) first ball is black and second is red (iii) one of them is black and other is red.

Ans: The total is 18 balls, so $P(\text{red}) = 8/18 = 4/9$ and $P(\text{black}) = 10/18 = 5/9$ on each draw (with replacement, the draws are independent). (i) $P(\text{both red}) = (4/9) \times (4/9) = 16/81$. (ii) $P(\text{1st black, 2nd red}) = (5/9) \times (4/9) = 20/81$. (iii) $P(\text{one black and one red}) = P(\text{black then red}) + P(\text{red then black}) = 20/81 + 20/81 = 40/81$.

14. Probability of solving a specific problem independently by A and B are $1/2$ and $1/3$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

Ans: $P(A \text{ solves}) = 1/2$, $P(B \text{ solves}) = 1/3$, and A and B work independently. (i) $P(\text{problem solved}) =$

$1 - P(\text{neither solves}) = 1 - (1 - 1/2)(1 - 1/3) = 1 - (1/2)(2/3) = 1 - 1/3 = 2/3$. (ii) $P(\text{exactly one solves}) = P(A \text{ solves}) \times P(B \text{ doesn't}) + P(A \text{ doesn't}) \times P(B \text{ solves}) = (1/2)(2/3) + (1/2)(1/3) = 1/3 + 1/6 = 1/2$.

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent? (i) E: the card drawn is a spade, F: the card drawn is an ace (ii) E: the card drawn is black, F: the card drawn is a king (iii) E: the card drawn is a king or queen, F: the card drawn is a queen or jack.

Ans: (i) $P(E) = 13/52 = 1/4$, $P(F) = 4/52 = 1/13$. There is exactly one ace of spades, so $P(E \cap F) = 1/52 = P(E) \times P(F)$. So E and F are independent. (ii) $P(E) = 26/52 = 1/2$, $P(F) = 4/52 = 1/13$. There are 2 black kings, so $P(E \cap F) = 2/52 = 1/26 = P(E) \times P(F)$. So E and F are independent. (iii) $P(E) = 8/52 = 2/13$, $P(F) = 8/52 = 2/13$. The only cards common to both are queens, so $P(E \cap F) = 4/52 = 1/13$. Since $P(E) \times P(F) = 4/169 \neq 1/13$, E and F are not independent.

16. In a hostel, 60% of the students read Hindi newspapers, 40% read English newspapers and 20% read both Hindi and English newspapers. A student is selected at random. (a) Find the probability that she reads neither Hindi nor English newspapers. (b) If she reads Hindi newspaper, find the probability that she reads English newspaper. (c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Ans: Let H = "reads Hindi" and E = "reads English". $P(H) = 0.6$, $P(E) = 0.4$, $P(H \cap E) = 0.2$. (a) $P(\text{neither}) = 1 - P(H \cup E) = 1 - [P(H) + P(E) - P(H \cap E)] = 1 - [0.6 + 0.4 - 0.2] = 1 - 0.8 = 0.2 = 1/5$. (b) $P(E|H) = P(H \cap E)/P(H) = 0.2/0.6 = 1/3$. (c) $P(H|E) = P(H \cap E)/P(E) = 0.2/0.4 = 1/2$.

17. A pair of dice is thrown. Find the probability of getting an even prime number on each die.

Ans: Choose the correct answer: (A) 0 (B) $1/3$ (C) $1/12$ (D) $1/36$. On a single die, the only even prime number is 2, so $P(2 \text{ on a die}) = 1/6$. Since the two dice are independent, $P(\text{even prime on each die}) = (1/6) \times (1/6) = 1/36$. Therefore the correct answer is (D) $1/36$.

18. Two events A and B will be independent if:

Ans: Choose the correct answer: (A) A and B are mutually exclusive (B) $P(A' \cap B') = [1 - P(A)][1 - P(B)]$ (C) $P(A) = P(B)$ (D) $P(A) + P(B) = 1$. Two events A and B are independent exactly when $P(A \cap B) = P(A)P(B)$, and taking complements this is equivalent to $P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A)P(B) = [1 - P(A)][1 - P(B)]$. Therefore the correct answer is (B) $P(A' \cap B') = [1 - P(A)][1 - P(B)]$.

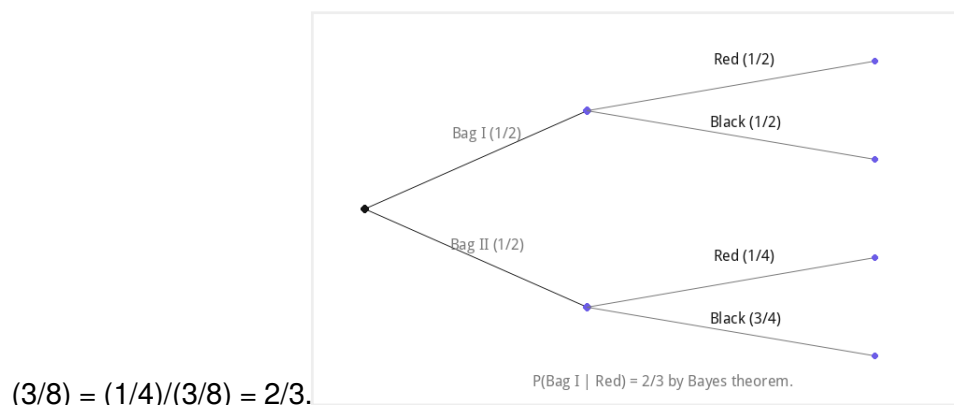
Exercise 13.3 Solutions (All 14 Questions)

1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn, and then a ball is drawn at random. What is the probability that the second ball is red?

Ans: If the first ball drawn is red (probability $5/10$), the urn then has 7 red and 5 black balls (12 total), so $P(2\text{nd red} | 1\text{st red}) = 7/12$. If the first ball drawn is black (probability $5/10$), the urn then has 5 red and 7 black balls (12 total), so $P(2\text{nd red} | 1\text{st black}) = 5/12$. By the law of total probability, $P(2\text{nd ball red}) = (5/10) \times (7/12) + (5/10) \times (5/12) = 7/24 + 5/24 = 12/24 = 1/2$.

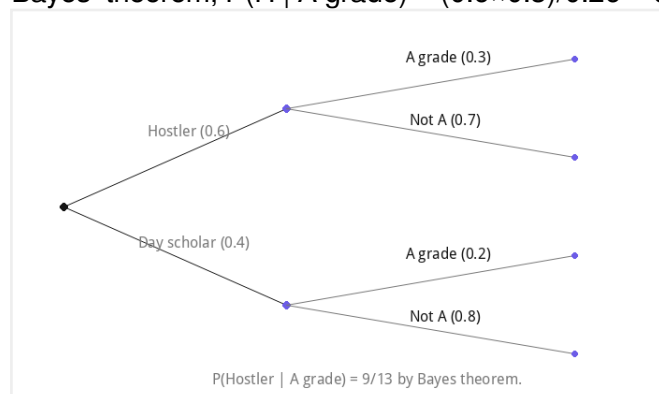
2. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Ans: Let Bag I have 4 red, 4 black and Bag II have 2 red, 6 black; each bag is equally likely to be chosen, so $P(\text{Bag I}) = P(\text{Bag II}) = 1/2$. $P(\text{red} | \text{Bag I}) = 4/8 = 1/2$, and $P(\text{red} | \text{Bag II}) = 2/8 = 1/4$. By the law of total probability, $P(\text{red}) = (1/2)(1/2) + (1/2)(1/4) = 1/4 + 1/8 = 3/8$. By Bayes' theorem, $P(\text{Bag I} | \text{red}) = [(1/2)(1/2)] /$



3. Of the students in a college, it is known that 60% reside in the hostel and 40% are day scholars. Previous year results report that 30% of all students who reside in the hostel attain A grade and 20% of day scholars attain A grade. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hostler?

Ans: Let H = “hostler” and D = “day scholar”, so $P(H) = 0.6$ and $P(D) = 0.4$. $P(\text{A grade} | H) = 0.3$ and $P(\text{A grade} | D) = 0.2$. By the law of total probability, $P(\text{A grade}) = 0.6 \times 0.3 + 0.4 \times 0.2 = 0.18 + 0.08 = 0.26$. By Bayes’ theorem, $P(H | \text{A grade}) = (0.6 \times 0.3) / 0.26 = 0.18 / 0.26 = 9/13$.

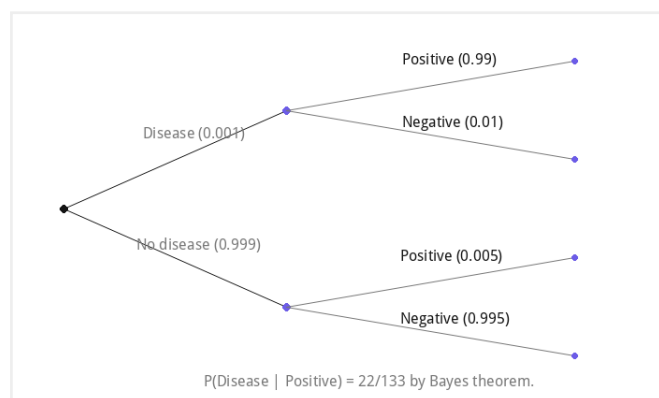


4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $3/4$ be the probability that he knows the answer and $1/4$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $1/4$, what is the probability that the student knew the answer to a question, given that he answered it correctly?

Ans: Let K = “knows the answer” and G = “guesses”, so $P(K) = 3/4$ and $P(G) = 1/4$. $P(\text{correct} | K) = 1$ and $P(\text{correct} | G) = 1/4$. By the law of total probability, $P(\text{correct}) = (3/4)(1) + (1/4)(1/4) = 3/4 + 1/16 = 12/16 + 1/16 = 13/16$. By Bayes’ theorem, $P(K | \text{correct}) = [(3/4)(1)] / (13/16) = (3/4) \times (16/13) = 12/13$.

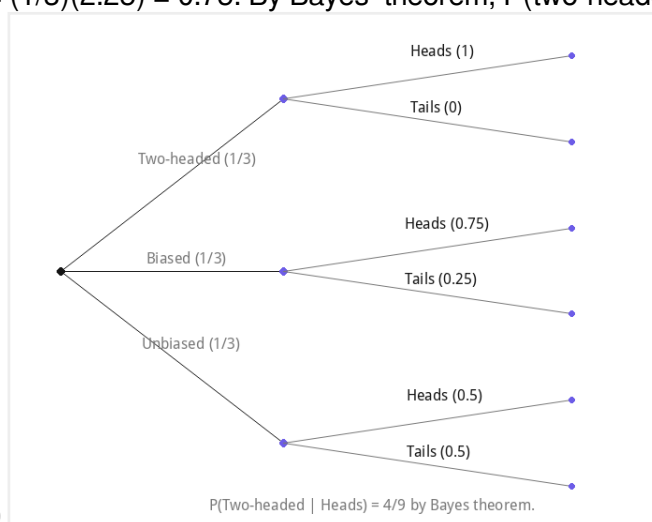
5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 0.5% of the healthy persons tested. If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that the test result is positive?

Ans: Let D = “has the disease”, so $P(D) = 0.001$ and $P(D') = 0.999$. $P(\text{positive} | D) = 0.99$ and $P(\text{positive} | D') = 0.005$. By the law of total probability, $P(\text{positive}) = 0.001 \times 0.99 + 0.999 \times 0.005 = 0.00099 + 0.004995 = 0.005985$. By Bayes’ theorem, $P(D | \text{positive}) = 0.00099 / 0.005985 = 22/133$ (approximately 0.165).



6. There are three coins. One is a two headed coin (having heads on both faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two headed coin?

Ans: Each coin is equally likely to be chosen, so each has probability $1/3$. $P(\text{heads} | \text{two-headed}) = 1$, $P(\text{heads} | \text{biased}) = 0.75$, $P(\text{heads} | \text{unbiased}) = 0.5$. By the law of total probability, $P(\text{heads}) = (1/3)(1) + (1/3)(0.75) + (1/3)(0.5) = (1/3)(2.25) = 0.75$. By Bayes' theorem, $P(\text{two-headed} | \text{heads}) =$



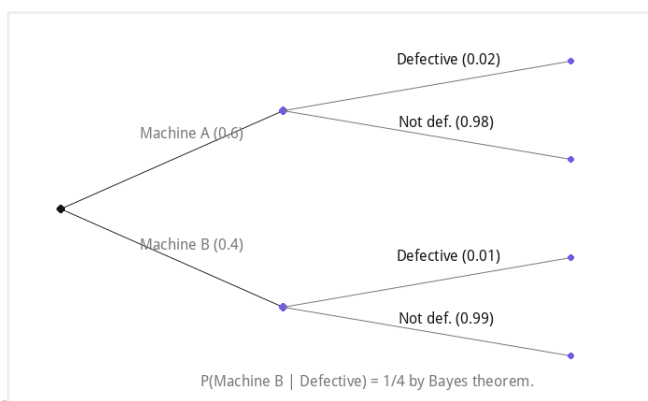
$$[(1/3)(1)]/0.75 = (1/3)/(3/4) = 4/9.$$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Ans: The total number of drivers is 12000, so $P(\text{scooter}) = 2000/12000 = 1/6$, $P(\text{car}) = 4000/12000 = 1/3$, $P(\text{truck}) = 6000/12000 = 1/2$. By the law of total probability, $P(\text{accident}) = (1/6)(0.01) + (1/3)(0.03) + (1/2)(0.15) = 0.001667 + 0.01 + 0.075 = 0.086667 = 13/150$. By Bayes' theorem, $P(\text{scooter} | \text{accident}) = [(1/6)(0.01)]/(13/150) = (1/600) \times (150/13) = 1/52$.

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Ans: $P(A) = 0.6$, $P(B) = 0.4$. $P(\text{defective} | A) = 0.02$, $P(\text{defective} | B) = 0.01$. By the law of total probability, $P(\text{defective}) = 0.6 \times 0.02 + 0.4 \times 0.01 = 0.012 + 0.004 = 0.016$. By Bayes' theorem, $P(B | \text{defective}) =$



$$(0.4 \times 0.01) / 0.016 = 0.004 / 0.016 = 1/4.$$

9. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7, and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Ans: $P(\text{Group 1 wins}) = 0.6$, $P(\text{Group 2 wins}) = 0.4$. $P(\text{new product} | \text{Group 1}) = 0.7$, $P(\text{new product} | \text{Group 2}) = 0.3$. By the law of total probability, $P(\text{new product}) = 0.6 \times 0.7 + 0.4 \times 0.3 = 0.42 + 0.12 = 0.54$. By Bayes' theorem, $P(\text{Group 2} | \text{new product}) = (0.4 \times 0.3) / 0.54 = 0.12 / 0.54 = 2/9$.

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Ans: $P(\text{die shows 5 or 6}) = 2/6 = 1/3$, and $P(\text{die shows 1, 2, 3 or 4}) = 4/6 = 2/3$. If 5 or 6, she tosses the coin thrice, so $P(\text{exactly 1 head} | 5 \text{ or } 6) = C(3, 1)(1/2)^3 = 3/8$. If 1, 2, 3 or 4, she tosses the coin once, so $P(\text{exactly 1 head} | 1, 2, 3 \text{ or } 4) = 1/2$. By the law of total probability, $P(\text{exactly 1 head}) = (1/3)(3/8) + (2/3)(1/2) = 1/8 + 1/3 = 3/24 + 8/24 = 11/24$. By Bayes' theorem, $P(1, 2, 3 \text{ or } 4 | \text{exactly 1 head}) = [(2/3)(1/2)] / (11/24) = (1/3) \times (24/11) = 8/11$.

11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced. What is the probability that it was produced by A?

Ans: $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$. $P(\text{defective} | A) = 0.01$, $P(\text{defective} | B) = 0.05$, $P(\text{defective} | C) = 0.07$. By the law of total probability, $P(\text{defective}) = 0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07 = 0.005 + 0.015 + 0.014 = 0.034$. By Bayes' theorem, $P(A | \text{defective}) = 0.005 / 0.034 = 5/34$.

12. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Ans: $P(\text{lost card is a diamond}) = 13/52 = 1/4$, and $P(\text{lost card is not a diamond}) = 39/52 = 3/4$. If the lost card is a diamond, the remaining 51 cards contain 12 diamonds, so $P(\text{draw 2 diamonds} | \text{lost is diamond}) = C(12, 2) / C(51, 2) = 66/1275$. If the lost card is not a diamond, the remaining 51 cards contain 13 diamonds, so $P(\text{draw 2 diamonds} | \text{lost is not diamond}) = C(13, 2) / C(51, 2) = 78/1275$. By the law of total probability, $P(\text{draw 2 diamonds}) = (1/4)(66/1275) + (3/4)(78/1275) = (16.5 + 58.5) / 1275 = 75/1275$. By Bayes' theorem, $P(\text{lost is diamond} | \text{draw 2 diamonds}) = [(1/4)(66/1275)] / (75/1275) = 16.5/75 = 11/50$.

13. Probability that A speaks truth is 4/5. A coin is tossed. A reports that a head appears. Find

the probability that actually there was a head.

Ans: Choose the correct answer: (A) $4/5$ (B) $1/2$ (C) $1/5$ (D) $2/5$. Let $T =$ "A speaks the truth" with $P(T) = 4/5$, so $P(\text{lies}) = 1/5$. The actual result is head with probability $1/2$ and tail with probability $1/2$. $P(\text{A reports head}) = P(\text{actual head}) \times P(\text{truth}) + P(\text{actual tail}) \times P(\text{lie}) = (1/2)(4/5) + (1/2)(1/5) = 2/5 + 1/10 = 4/10 + 1/10 = 1/2$. By Bayes' theorem, $P(\text{actual head} | \text{reports head}) = [(1/2)(4/5)] / (1/2) = 4/5$. Therefore the correct answer is (A) $4/5$.

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

Ans: Choose the correct answer: (A) $P(A|B) = P(B)/P(A)$ (B) $P(A|B) < P(A)$ (C) $P(A|B) \geq P(A)$ (D) None of these. Since $A \subset B$, we have $A \cap B = A$, so $P(A|B) = P(A \cap B)/P(B) = P(A)/P(B)$. Since $P(B) \leq 1$, dividing $P(A)$ by a number at most 1 gives a result at least as large as $P(A)$, i.e. $P(A)/P(B) \geq P(A)$. Therefore the correct answer is (C) $P(A|B) \geq P(A)$.

Miscellaneous Exercise Solutions (All 13 Questions)

1. A and B are two events such that $P(A) \neq 0$. Find $P(B|A)$ if (i) A is a subset of B (ii) $A \cap B = \emptyset$.

Ans: (i) If $A \subset B$, then $A \cap B = A$, so $P(B|A) = P(A \cap B)/P(A) = P(A)/P(A) = 1$. (ii) If $A \cap B = \emptyset$, then $P(A \cap B) = 0$, so $P(B|A) = P(A \cap B)/P(A) = 0/P(A) = 0$.

2. A couple has two children. (i) Find the probability that both children are males, if it is known that at least one of the children is male. (ii) Find the probability that both children are females, if it is known that the elder child is a female.

Ans: The sample space (elder, younger) is $\{BB, BG, GB, GG\}$, each equally likely. (i) $F =$ "at least one is male" $= \{BB, BG, GB\}$, and $E =$ "both male" $= \{BB\}$, so $E \cap F = \{BB\}$. Hence $P(E|F) = (1/4)/(3/4) = 1/3$. (ii) $F =$ "elder is female" $= \{GB, GG\}$, and $E =$ "both female" $= \{GG\}$, so $E \cap F = \{GG\}$. Hence $P(E|F) = (1/4)/(2/4) = 1/2$.

3. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male, assuming that there are equal numbers of males and females?

Ans: $P(\text{male}) = P(\text{female}) = 1/2$. $P(\text{grey hair}|\text{male}) = 0.05$, $P(\text{grey hair}|\text{female}) = 0.0025$. By the law of total probability, $P(\text{grey hair}) = (1/2)(0.05) + (1/2)(0.0025) = 0.025 + 0.00125 = 0.02625$. By Bayes' theorem, $P(\text{male}|\text{grey hair}) = 0.025/0.02625 = 20/21$.

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Ans: Let $p = P(\text{right-handed}) = 0.9$ and $q = 0.1$, with $n = 10$ independent trials. The probability of at most 6 right-handed people is $P(X \leq 6) = 1 - P(X \geq 7)$, where $P(X \geq 7)$ is the sum of the individual binomial probabilities $C(10, r) \times (0.9)^r \times (0.1)^{10-r}$ for $r = 7, 8, 9, 10$. Explicitly, $P(X \leq 6) = 1 - [C(10, 7)(0.9)^7(0.1)^3 + C(10, 8)(0.9)^8(0.1)^2 + C(10, 9)(0.9)^9(0.1) + C(10, 10)(0.9)^{10}] = 1 - [120(0.9)^7(0.1)^3 + 45(0.9)^8(0.1)^2 + 10(0.9)^9(0.1) + (0.9)^{10}]$, which works out to approximately 0.0128.

5. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Ans: A leap year has 366 days $= 52$ complete weeks $+ 2$ extra days. These 2 extra days can be any one of the 7 equally likely pairs: (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun). A leap year has 53 Tuesdays exactly when the extra 2 days include a Tuesday, which happens for 2 of these 7 pairs: (Mon, Tue) and (Tue, Wed). So the required probability is $2/7$.

6. There are four boxes A, B, C, D containing coloured marbles as follows — Box A: 1 red, 6 white, 3 black; Box B: 6 red, 2 white, 2 black; Box C: 8 red, 1 white, 1 black; Box D: 0 red, 6 white, 4 black (10 marbles in each box). One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A? Box B? Box C?

Ans: Each box is equally likely to be chosen, so $P(A) = P(B) = P(C) = P(D) = 1/4$. $P(\text{red}|A) = 1/10$, $P(\text{red}|B) = 6/10$, $P(\text{red}|C) = 8/10$, $P(\text{red}|D) = 0/10$. By the law of total probability, $P(\text{red}) = (1/4)(1/10 + 6/10 + 8/10 + 0) = (1/4)(15/10) = 3/8$. By Bayes' theorem: $P(A|\text{red}) = [(1/4)(1/10)]/(3/8) = (1/40) \times (8/3) = 1/15$. $P(B|\text{red}) = [(1/4)(6/10)]/(3/8) = (6/40) \times (8/3) = 2/5$. $P(C|\text{red}) = [(1/4)(8/10)]/(3/8) = (8/40) \times (8/3) = 8/15$.

7. Assume that the chances of a patient having a heart attack are 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of a certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

Ans: $P(\text{yoga}) = P(\text{drug}) = 1/2$. $P(\text{heart attack}|\text{yoga}) = 0.4 \times (1 - 0.3) = 0.28$, and $P(\text{heart attack}|\text{drug}) = 0.4 \times (1 - 0.25) = 0.3$. By the law of total probability, $P(\text{heart attack}) = (1/2)(0.28) + (1/2)(0.3) = 0.14 + 0.15 = 0.29$. By Bayes' theorem, $P(\text{yoga}|\text{heart attack}) = 0.14/0.29 = 14/29$.

8. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the four entries of the determinant are chosen independently, each being equally likely to be 0 or 1.)

Ans: There are $2^4 = 16$ equally likely determinants $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ with a, b, c, d each 0 or 1, and the determinant's value is $ad - bc$. Checking all 16 combinations, $ad - bc$ is positive (equals 1) only when $a=1, d=1$ and at least one of b, c is 0, i.e. $(b, c) \neq (1, 1)$; this happens for 3 of the 4 possible (b, c) pairs, giving 3 favourable determinants out of 16. So the required probability is $3/16$.

9. An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities are assumed to be known: $P(A \text{ fails}) = 0.2$, $P(B \text{ fails alone}) = 0.15$, $P(A \text{ and } B \text{ fail}) = 0.15$. Evaluate the following probabilities: (i) $P(A \text{ fails}|B \text{ has failed})$ (ii) $P(A \text{ fails alone})$.

Ans: $P(B \text{ fails}) = P(B \text{ fails alone}) + P(A \text{ and } B \text{ fail}) = 0.15 + 0.15 = 0.30$. (i) $P(A \text{ fails}|B \text{ failed}) = P(A \text{ and } B \text{ fail})/P(B \text{ fails}) = 0.15/0.30 = 1/2$. (ii) $P(A \text{ fails alone}) = P(A \text{ fails}) - P(A \text{ and } B \text{ fail}) = 0.2 - 0.15 = 0.05$.

10. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Ans: $P(\text{transferred red}) = 3/7$, $P(\text{transferred black}) = 4/7$. If the transferred ball is red, Bag II has 5 red and 5 black (10 total), so $P(\text{draw red}|\text{transferred red}) = 5/10 = 1/2$. If the transferred ball is black, Bag II has 4 red and 6 black (10 total), so $P(\text{draw red}|\text{transferred black}) = 4/10 = 2/5$. By the law of total probability, $P(\text{draw red}) = (3/7)(1/2) + (4/7)(2/5) = 3/14 + 8/35 = 15/70 + 16/70 = 31/70$. By Bayes' theorem, $P(\text{transferred black}|\text{draw red}) = [(4/7)(2/5)]/(31/70) = (8/35) \times (70/31) = 16/31$.

11. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then which of the following is true?

Ans: Choose the correct answer: (A) $A \subset B$ (B) $B \subset A$ (C) $B = \emptyset$ (D) $P(B) = 1$. Since $P(B|A) = P(A \cap B)/P(A) = 1$, we get $P(A \cap B) = P(A)$. Since $A \cap B \subseteq A$ always holds, and here $P(A \cap B)$ equals $P(A)$, this forces $A \cap B = A$, which means $A \subset B$. Therefore the correct answer is (A) $A \subset B$.

12. If $P(A|B) > P(A)$, then which of the following is correct?

Ans: Choose the correct answer: (A) $P(B|A) < P(B)$ (B) $P(A \cap B) < P(A) \cdot P(B)$ (C) $P(B|A) > P(B)$ (D) $P(B|A) = P(B)$. From $P(A|B) > P(A)$, we get $P(A \cap B)/P(B) > P(A)$, so $P(A \cap B) > P(A) \cdot P(B)$. Dividing both sides by $P(A)$, $P(A \cap B)/P(A) > P(B)$, i.e. $P(B|A) > P(B)$. Therefore the correct answer is (C) $P(B|A) > P(B)$.

13. If A and B are two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then which of the following is correct?

Ans: Choose the correct answer: (A) $P(B|A) = 1$ (B) $P(A|B) = 1$ (C) $P(B|A) = 0$ (D) $P(A|B) = 0$. From $P(A)+P(B)-P(A \cap B) = P(A)$, we get $P(B) = P(A \cap B)$. Therefore $P(A|B) = P(A \cap B)/P(B) = P(B)/P(B) = 1$ (assuming $P(B) \neq 0$). Therefore the correct answer is (B) $P(A|B) = 1$.

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